SS Cygni—A Nonlinear Look at 100 Years of AAVSO Data

N. Jevtic
Physics Department, University of Connecticut, 2152 Hillside Rd. U-3046, Storrs, CT 06269

Janet A. Mattei
AAVSO, 25 Birch Street, Cambridge, MA 02138

J. S. Schweitzer
Physics Department, University of Connecticut, 2152 Hillside Rd. U-3046, Storrs, CT 06269

Presented at the 91st Annual Meeting of the AAVSO, October 2002

Abstract A full nonlinear time series analysis was conducted of ~100 years of visual observations from the American Association of Variable Star Observers (AAVSO) International Database for the cataclysmic variable SS Cygni. Intensity was obtained from the AAVSO magnitude data using: $F(L) = 10^{-(0.4\,(\text{visual magnitude}) + 8.43)}$. Intensity is a "better" dynamical variable because energy couples all the active degrees of freedom. An intensity phase space portrait was reconstructed in 3-D using time delay coordinates. A series of Poincare sections was obtained. The Poincare section return times suggest a long-term periodicity of ~52 years. We also obtained the times between outbursts in a more traditional manner, the outbursts being defined with respect to a baseline determined by the adjacent quiescent sections. Excellent agreement was found between these two approaches. An observed region of discrepancy is for an interval containing many lower amplitude shorter outbursts. Earlier reports using the O–C method indicated a long-term variation on the order of at least 100 years, thus it remains to be seen whether we are detecting a fundamental frequency or its first harmonic.

1. Introduction

Observations of the brightness of a variable star over time result in a time series. This is our sole source of information about the dynamics of the star. We would like to learn as much as possible from this information. In addition to inspecting the variation for any features, we would like to know what periodicities are present. For this reason, usually the next step in the analysis of magnitude (or intensity) data is a Fourier analysis. A power spectrum is a linear tool. If we generate a sine wave for each line in the spectrum with the right amplitude and phase, by adding them all up, we should be able to reproduce our light curve. This works very well if there are only a few frequencies present. If, on the other hand, we get a broadband power spectrum with many frequencies, the usefulness of the power spectrum is very limited. Until recently, very little could be done in the case of broadband spectra that are obtained for many variable stars. However, with the computing power available today, nonlinear time series analysis, originally developed to study chaotic systems, can be used to study variable stars with broadband power spectra. One such nonlinear time series method is phase space reconstruction using time delay coordinates. This method is based on the Takens-Mane (Takens 1981; Mane 1981) theorem. Initially, the theorem was limited to systems governed by autonomous systems of equations. Fortunately, it has recently been extended to driven systems (Stark 1999) and most recently to stochastically driven systems (Stark 2003).

The work presented here on SS Cygni is just part of an effort (see for example Jevtic 2003; Jevtic *et al.* 2001; Jevtic *et al.* 2004) to explore the usefulness of these methods in astrophysics.

2. SS Cygni

The cataclysmic variable SS Cygni, a dwarf nova, is a close binary that consists of a primary white dwarf and a secondary red dwarf. The stars are thought to be separated by only 100,000 miles or less. Their orbital revolution period is ~6.5 h. The system is quiescent about 75% of the time (Cannizzo and Mattei 1992; Mattei *et al.* 1985, 1991, 1996). From the low state, it brightens without warning and usually reaches maximum in a time on the order of a day (for normal outbursts; anomalous outbursts can take up to 10 days to reach maximum). A seemingly random distribution of wide and narrow outbursts is observed. Outbursts recur every 4–10 weeks and last 7–18 days.

2.1. Data

About 38,500 one-day means obtained from the 220,000 individual brightness measurements submitted to the AAVSO International Database since 1896—or over 100 years of data—contributed by 2,014 observers worldwide were used for this study. These one-day mean magnitudes were converted to intensity (flux) using equation (1):

$$F(L) = 10^{-(0.4\ (\text{visual magnitude})\ +\ 8.43)} \tag{1}$$

The light intensity curve consists of about 800 outbursts. A section of the data is shown in Figure 1 with the characteristic sequence of wide and narrow outbursts.

The power spectrum for SS Cygni (Figure 2) is broadband, with no dominant periodicities. The ultimate goal of any time series analysis is to obtain information about the dynamics at the source of a signal and broadband spectra such as the one in Figure 2 cannot be explained using traditional, linear analysis methods. Resorting to nonlinear time series analysis is natural in these cases.

3. The method

We usually observe a system as a time series (in the time domain). Then we obtain its power spectrum (observe it in the frequency domain). In nonlinear time series analysis we transform our data into phase space vectors (Hegger *et al.* 1999)

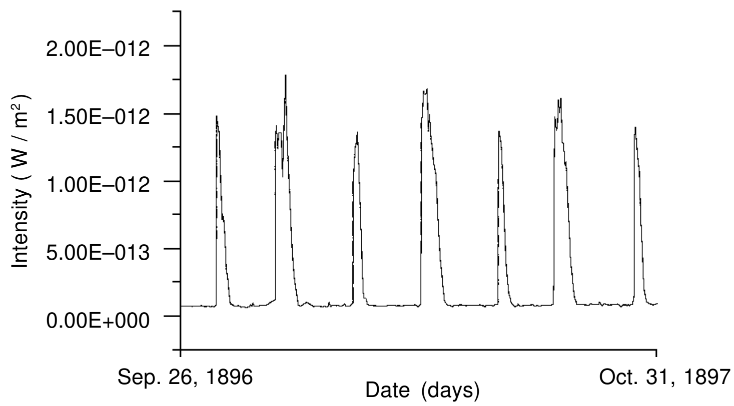

Figure 1. Section of intensity light curve.

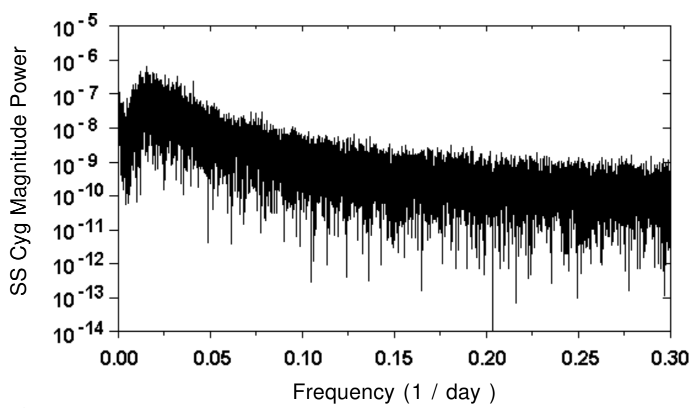

Figure 2. Power spectrum.

and observe the system in phase space to access nonlinear information. A phase space is obtained by going to coordinates that do not depend explicitly on time. The Takens-Mane theorem and its two Stark extensions guarantee that a time delay representation is a diffeomorphism for the original system whose dynamical properties can yield valuable information about the source of the signal. We use the time delay method of phase space reconstruction. If we measure a time series s (signal) with interval t, i.e. at time t, 2t, 3t, …

$$s(t), s(2t), s(3t), s(4t), \ldots \tag{2}$$

using time delay coordinates this uniformly sampled time series (2) yields a vector representation. At time T, instead of time series (2) we obtain the vector:

$$\mathbf{a}_T = \mathbf{a}(s(T), s(T+\tau), s(T+2\tau), s(T+3\tau), \ldots s(T+(m-1)\tau) \tag{3}$$

where time T is some multiple of the sampling time, m is the embedding dimension of our reconstructed phase space, and τ is the time delay. It is called a reconstructed phase space because we do not know *a priori* the equations that govern the dynamics at the source of the signal. However, the resulting phase space reconstruction is as if these equations were actually used to generate it (it is diffeomorphic). Thus, this phase space object can be used to explore various aspects of the dynamics of the real system. Ultimately, we hope to use the signal itself to obtain the equations governing the process at the source.

We consider as an example time series s(t):

$$1, 3, 6, 7, 4, 2, 4, 5, 6, \ldots \tag{4}$$

If we want to transform it into three dimensional vectors and use a delay of 1 we obtain the following vectors:

$$\begin{array}{ll} \mathbf{a}_1(1, 3, 6) & \\ \mathbf{a}_2(3, 6, 7) & \\ \mathbf{a}_3(6, 7, 4) & \text{etc.} \\ \mathbf{a}_4(7, 4, 2) & \end{array}$$

The delay is chosen such that the measurements, since they now function as components of vectors, are as independent as possible. In this work the optimal delay is chosen at a minimum of AMI, the average mutual information (Fraser and Swinney 1986). In phase space time is an implicit variable. Time information may be recouped using Poincare sections. In three dimensions, a Poincare section is a suitably positioned plane (Figure 3). Of interest are the distributions of points A and B at which the trajectory intersects this plane and the time it takes for one circuit,

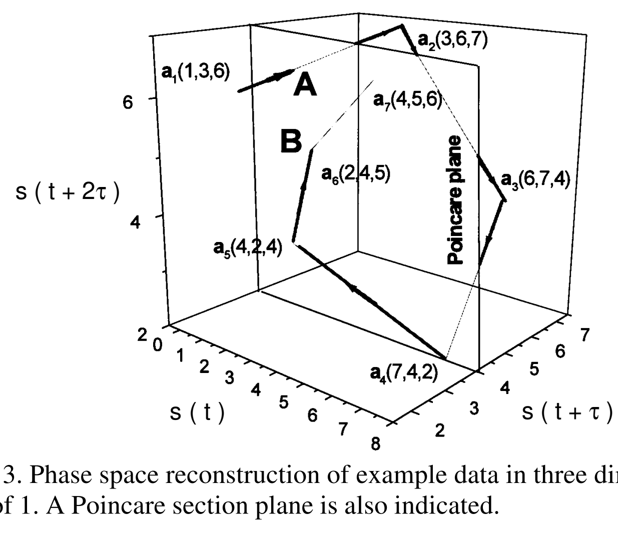

Figure 3. Phase space reconstruction of example data in three dimensions with a delay of 1. A Poincare section plane is also indicated.

in our case the time between two successive intersections with the plane at points A and B.

In the case of SS Cygni recurrence plots were used to explore stationarity. These plots are suggestive that most, if not all, of the time series is stationary. Therefore, we proceed as if stationarity was demonstrated and that 38,500 points is a long enough data set. The data are uniformly sampled once every day and gaps are linearly interpolated. The oldest most unevenly sampled data have been excluded from the analysis with the data set starting on Sept. 26, 1896. The range of the magnitude data, however, is not overly large and these data are in effect "coarsely digitized," which has the effect of poor resolution. Both of these drawbacks are somewhat mitigated when the conversion to intensity is made.

4. Nonlinear intensity light curve analysis

We analyze the quasi-periodic light curve of SS Cygni (a section of which is shown in Figure 1) treating SS Cygni as a nonlinear deterministic system. We use time delay coordinates and obtain the phase-space representation shown in Figure 4 in an attempt to probe the nonlinear behavior of this system.

For noisy data the traditional methods for estimating the reconstruction dimension become unreliable. We embed the SS Cygni data in three dimensions as initial tests on the data indicated that the Poincare section return times in higher phase-space dimensions do not change. In higher dimensions we obtain the same Poincare section return time curves as those for three dimensions. We use a delay τ of 40 days that was obtained at the minimum of average mutual information (Fraser and Swinney 1980). This delay ensures the greatest degree of coordinate independence.

From the magnitude data, using Equation 1, we obtain light intensity curves. For a delay of 40 days the phase-space reconstruction is shown in Figure 4. For clarity, only the phase space points are depicted. In the inset we show the actual trajectory for the section of the data shown in Figure 1. The orientation of a Poincare plane is also shown.

5. Poincare section

Earlier work (Hempelmann and Kurths 1990) using the O–C (observed minus calculated) diagram identified a secular variation of at least on the order of 100 years.

We use Poincare section return times to evaluate the variation of the inter-outburst interval. The Poincare section used is a plane parallel to the $s(t) - s(t + 40d)$ plane at the average value of the data (Figure 4, inset). The Poincare section return time curve is quasi-periodic with a period that appears to be about 52 years (Figure 5).

In addition, the time interval between successive outbursts (the vertical axis) was determined using adjacent quiescent sections of the light curve as a baseline (Figure 5, solid curve) and determining the position of the maximum by fitting each

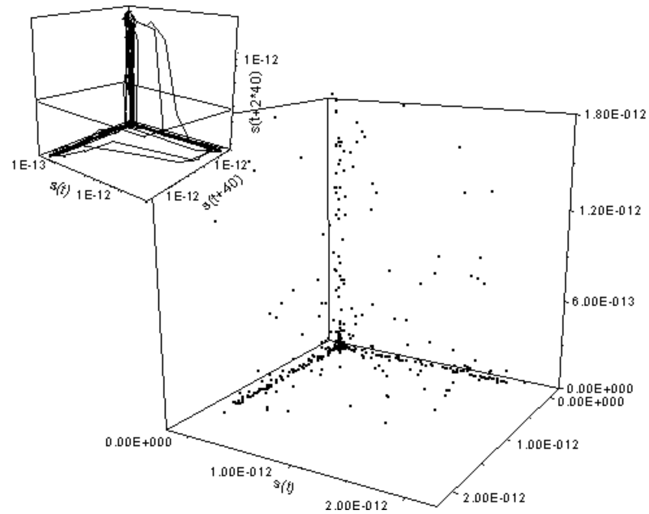

Figure 4. Phase space reconstruction in three dimensions (delay = 40 days) of SS Cygni intensity data. For clarity, only the end points of the delay vectors are shown. In the inset, we show the phase-space reconstruction for the section of the data shown in Figure 1.

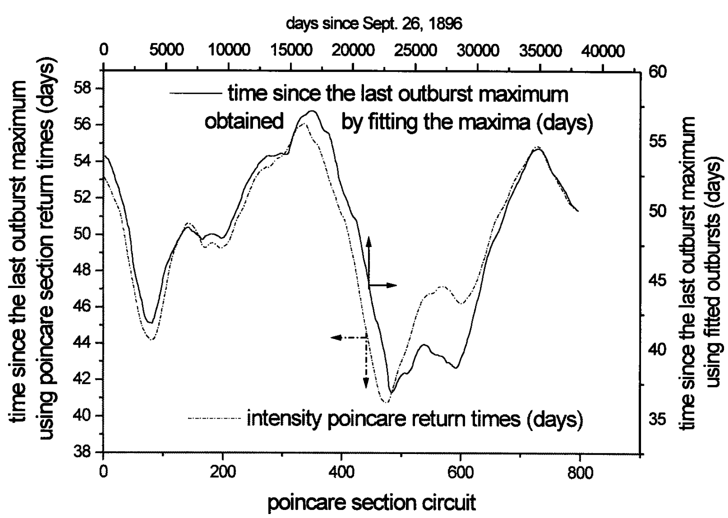

Figure 5. Comparison of the time between successive outburst maxima (solid) and Poincare section return times (dot dash). The curves are for ~ 100 years of data. The top horizontal axis is time in days since Sept. 26, 1896. The bottom horizontal axis is the phase space circuit or cycle number and corresponds to the outburst number. The vertical axis is the time since the last outburst. Four hundred cycles correspond to approximately 52 years which may reflect the fundamental frequency or its harmonic.

outburst with a Gaussian. The time variation of this curve is in good agreement with that of the Poincare section return times (Figure 5, dashed curve). Both methods suggest a periodicity of ~52 years. Hempelmann and Kurths (1990), using the O–C diagram (Lombard and Koen 1993), report a cycle length time scale on the order of at least 100 years. While our results are most suggestive, a definitive determination of the period has to wait for the collection of data over a longer period.

6. Conclusions and further work

The determination of the time between successive outburst maxima using Poincare section return times suggests that there may be a 52-year periodicity. The time between outbursts was also determined using traditional Gaussian fitting of the outbursts. The agreement between the Poincare section return time curve and that for the traditionally determined time difference between the maxima of the outbursts is good. The region of the greatest difference between the two methods is between outbursts 480 and 600. This region is populated by many more shorter and lower amplitude outbursts and it is these shorter, lower amplitude outbursts that account for the difference. Earlier reports using the O–C method were of a long-term variation on the order of at least 100 years (Hemplemann and Kurths 1990), though our current results indicate a period of about half that. This period remains to be confirmed over time. It also remains to be determined whether we are detecting the fundamental frequency or its first harmonic. However, the utility of nonlinear time series analysis for detecting hidden periodicities in signals with continuous power spectra has been demonstrated and should be extended to more U Geminorum systems and other types of variable stars.

7. Acknowledgements

The analysis presented here was conducted using TISEAN software (Hegger *et al.* 1999).

References

Cannizzo, J. K., and Mattei, J. A. 1992, *Astrophys. J.*, **401**, 642.
Fraser, A. M., and Swinney, H. L. 1986, *Phys. Rev. A*, **33**, 1134.
Hegger, R., Kantz, H., and Schreiber, T. 1999, "Practical Implementation of Nonlinear Time Series Methods: The TISEAN Package" (software), *CHAOS,* **9**, 413.
Hemplemann, A., and Kurths, J. 1990, *Astron. Astrophys.*, **232**, 356.
Jevtic, N. 2003, Nonlinear time series analysis of solar and stellar data, Ph.D. Thesis, University of Connecticut.
Jevtic, N., Zelechoski, S., Feldman, H., Petersen, C., and Schweitzer, J. S. 2004, "Nonlinear time series analysis of PG1351+489 light intensity curves," submitted to *Astrophys. J.*

Jevtic N., Schweitzer, J. S., and Cellucci, C. J. 2001, *Astron. Astrophys.*, **379**, 611.

Lombard, F., and Koen, C. 1993, *Mon. Not. Royal Astron. Soc.*, **263**, 309.

Mattei, J. A., Saladyga, M., Waagen, E. O., and Jones, C. M. 1985, *AAVSO Monograph 1, AAVSO, SS Cygni Light Curves 1896–1985*, Cambridge, MA.

Mattei, J. A., Waagen, E. O., and Foster, E. G. 1991, *AAVSO Monograph 1, Supplement 1, SS Cygni Light Curves 1985–1990*, AAVSO, Cambridge, MA.

Mattei, J. A., Waagen, E. O., and Foster, E. G. 1996, *AAVSO Monograph 1, Supplement 2, AAVSO, SS Cygni Light Curves 1991–1995*, Cambridge, MA.

Mane, R. 1981, "On the dimension of compact invariant sets of certain non-linear maps," in D. A. Rand and L.-S. Young , eds., *Dynamical Systems and Turbulence*, Warwick 1980, Springer-Verlag, Berlin, 230.

Stark, J. 1999, *J. of Nonlinear Science*, **9**, 255.

Stark, J, Broomhead, D. S., Davies, M. E., and Huke, J. 2003, *J. Nonlinear Science*, **13**, 519.

Takens, F. 1981, "Detecting strange attractors in turbulence," in D. A. Rand and L-S. Young, eds., *Dynamical Systems and Turbulence*, Warwick 1980, Springer-Verlag, Berlin, 366–381.