WAVELET ANALYSIS OF SMALL-AMPLITUDE PULSATING RED GIANTS

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Abstract

We have investigated the usefulness of wavelet analysis for studying the changing period and amplitude of small-amplitude pulsating red giants. Specifically, we have applied it to EU Del, W Boo, and SX UMi. With care, this method can provide useful information about variables with amplitudes between 0.2 and one magnitude, especially if used in conjunction with light curves, Fourier analysis, and autocorrelation analysis.

1. Introduction

It is now apparent that all red giant stars, cooler than spectral type K5 III, are variable in brightness. The K5–M0 III stars tend to have *V* amplitudes of up to 0.1 magnitude; the M0–M5 III stars tend to have *V* amplitudes of 0.1 to 1 magnitude. Only the rare late M giants—the Mira stars—have amplitudes of several magnitudes; their variability is easily apparent to visual observers.

The variability of the M0–M5 III stars has been extensively studied on time scales of a decade or more, both through the AAVSO Photoelectric Photometry program (Percy *et al.* 1996, 2001a), and using robotic telescopes (Percy *et al.* 2001b). These studies have shown that: (1) the incidence and amplitude of variability increase with decreasing temperature; (2) most stars show one or two periods, in the range 10–200 days, which are probably due to radial pulsation; (3) the variability tends to be semiregular at best: individual cycles vary in amplitude and light curve shape, and it is not clear whether this is due to multi-periodicity or irregularity; (4) in a few stars, the dominant pulsation mode may switch from one period to another; (5) some stars also vary on a time scale which is an order of magnitude longer than the ones mentioned in (2), and the nature of this slow variability is not known.

Until now, we have used a combination of light curve analysis, Fourier (power spectrum) analysis, and autocorrelation analysis to study these stars.

Wavelet analysis is a more sophisticated technique which is useful for tracking the changing period(s) and amplitudes of variable stars. It is similar to Fourier analysis, but it can examine the power spectrum of limited time spans of data with greater ease. This method has already been used to study large-amplitude and medium-amplitude pulsating red giants (*e.g.*, Bedding *et al.* 1998; Hawkins *et al.* 2001; Szatmáry *et al.* 1994). The goal of the present project is to test this technique on small-amplitude pulsating red giants (SAPRGs), and, if possible, obtain some useful scientific results. These stars present several technical challenges: smaller amplitudes, more irregularity, and significant seasonal gaps in the measurements. Specifically, we address three scientific questions: (1) the period and amplitude of EU Del—a well-studied member of this class—and their possible variations; (2) possible pulsation mode switching in the very small-amplitude variable W Boo; and (3) the period behavior of the circumpolar variable SX UMi which had no seasonal gaps.

2. Stars and data

EU Del (HR 7886, SpT M6III, $V \sim 6.25$) has a total range of about one magnitude in *V*, and a period of 62.5 days (Percy *et al.* 1996, 2001b); it can be considered the "prototype" of its class. We have re-analyzed *V* measurements (JD 2445500 to 2451500) from the AAVSO Photoelectric Photometry program (Landis 2000). We also analyzed five-day means of AAVSO visual measurements (JD 2440000 to 2451500), kindly provided by Dr. Janet A. Mattei (Mattei 2001). The latter measurements are less accurate than the photoelectric photometry measurements, but there are more of them, and the problem of the seasonal gap is less severe.

W Boo (HR 5490, SpT M3III, $V \sim 4.81$) has a total range in *V* of about 0.25 magnitude. It was suspected of mode-switching on the basis of Fourier analysis of individual seasons of measurements between 1985 and 1994 (Percy *et al.* 1996; Percy and Desjardins 1996). We have re-analyzed *V* measurements (JD 2445500 to 2451000) in the AAVSO Photoelectric Photometry program (Landis 2000).

SX UMi (HD 126409, SpT M, $V \sim 8.00$) is a circumpolar pulsating red giant with a total range in *V* of about 0.2 magnitude. The *V* measurements (JD 2450500 to 2451300) were kindly provided by Josep M. Gomez Forrellad and Enrique Garcia Melendo for another study (Percy *et al.* 2001c).

3. Analysis

Measurements were analyzed using the AAVSO's on-line (www.aavso.org) WWZ/WWA wavelet analysis software, following the on-line instructions and published description of the method (Foster 1996). The software was first applied (successfully) to the on-line test data, and then to the measurements of the small-amplitude pulsating red giants. The user-controlled variable "c," which specifies the trade-off between time resolution and frequency resolution, was set to 0.005 (Bedding *et al.* 1998).

4. Results—EU Del

Figure 1 shows the most significant period of EU Del, from AAVSO Photoelectric Photometry measurements, as a function of time. The well-known 62.5-day period (Percy *et al.* 1996, 2001b) is usually dominant but, at two epochs, a 77±2-day period appears to be dominant. The first epoch (JD 2448300–2448800) includes one season and the two adjacent seasonal gaps; the second epoch (JD 2450150–2451000) includes two seasons and three seasonal gaps. Inspection of the light curves shows that the 77±2-day period is an acceptable fit, but the simplest interpretation is that the 77±2-day period is an artifact of the irregularity of the star; the fact that the 77±2-day period is also an alias of the 62.5-day period means that it will also bridge the seasonal gaps—even if the 62.5-day period is actually the correct one. (Alias periods occur when there are regular gaps in the data, such as seasonal gaps; they occur because the number of cycles which occur during the gaps is uncertain. Alias periods due to seasonal gaps are related to the true period by 1/(alias period) = 1/(true period) ± n/365.25 where n is a small integer, so alias periods of 62.5 days include 75.4 days (n = -1) and 95.1 days (n = -2).)

A less probable interpretation is that the period of the star truly changes to 78 days during these epochs. Note that the 62.5-day period has been unambiguously confirmed as the basic period of the star by autocorrelation analysis, which is not affected by alias periods (Percy *et al.* 1996, 2001b). Based on two decades of experience with this and similar stars, we strongly suspect that the irregularity, together with the gaps, is responsible for these results.

Figure 2 shows the most significant period of EU Del, from 5-day means of AAVSO visual measurements, as a function of time. Again, the 62.5-day period is almost always dominant. This time, however, a 95±5-day period is occasionally seen. This period is a 2-cycle-per-year alias of the 62.5-day period. There are no seasonal gaps in the visual means, but there are corresponding times when the data are sparse. The measurements may also be affected by position-angle effects when the star moves from the evening to the morning sky, as we have found with P Cygni (Percy *et al.* 2001d).

A plot of the amplitude *vs.* time graph of EU Del shows some interesting results: (1) the amplitude was unusually small between JD 2450500 and 2451000; this epoch was one when the 77±2-day period appeared to be dominant; (2) during the first epoch when the 77±2-day period was most significant, and at the beginning of the second epoch, the amplitude of the 77±2-day period was only marginally higher than that of the 62.5-day period; and (3) prior to the low-amplitude epoch, the overall amplitude varied conspicuously on a time scale of about 500 days. The cause of the amplitude variation is not known.

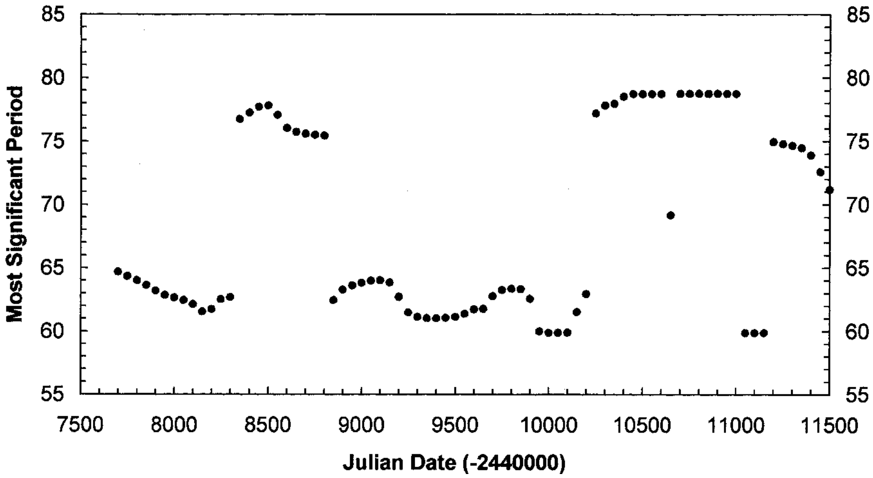

Figure 1. The most significant period of EU Del, as a function of time, based on wavelet analysis of AAVSO Photoelectric Photometry data. The 62.5-day known period is usually dominant. The 77±2-day period is probably an artifact of irregularity and seasonal gaps, as explained in the text.

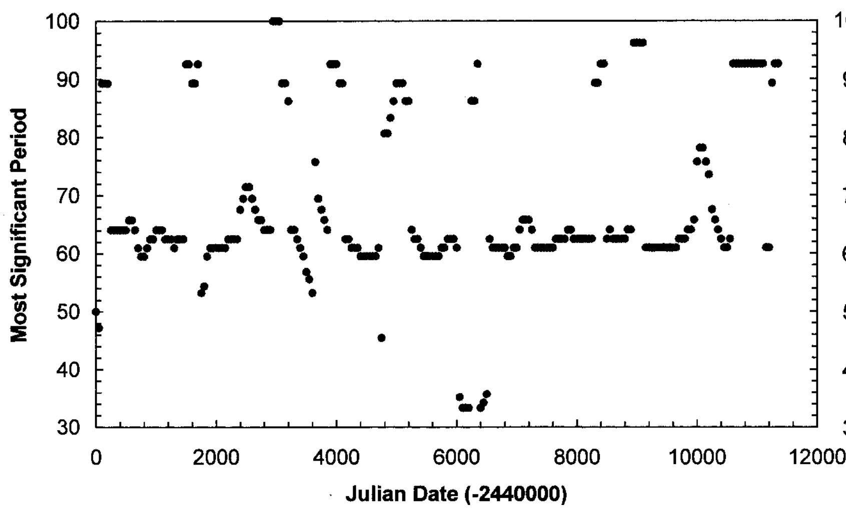

Figure 2. The most significant period of EU Del, as a function of time, based on wavelet analysis of 5-day means of AAVSO visual data. The 62.5-day known period is usually dominant. The 95±5-day period is probably an artifact of irregularity, systematic error, and sparse data during seasonal gaps, as explained in the text.

5. Results—W Boo

Figure 3 shows the most significant period of W Boo, from AAVSO Photoelectric Photometry measurements, as a function of time. The 25-day period is usually dominant but (even though the rest of Figure 3 may look like a scatter diagram), a histogram of the number of occurrences of the most significant period shows clear peaks at 25 days, 35±3 days, and 55±3 days. These are consistent with the results of the Fourier analysis (Percy and Desjardins 1996), and with the presence of three different low-order radial pulsation modes in the star (Percy and Parkes 1998).

There appear to be changes in the amplitude on a time scale of 500 to 1000 days but, given the complexity and small amplitude of the star, it is not possible to derive firm conclusions.

6. Results—SX UMi

Figure 4 shows the most significant period of SX UMi, as a function of time, from CCD photometry provided by Josep M. Gomez Forrellad and Enrique Garcia Melendo. Except for one brief epoch, the dominant period is about 38 days. A similar value has been determined by autocorrelation analysis of the same data (Percy *et al.* 2001d). The light curve is far from periodic, and the semiregularity could be explained by low-amplitude secondary periods, or irregularity. These could cause the underlying 38-day period to appear to "wander" as it does in Figure 4. Alternatively, the apparent change of period could be real.

7. Discussion and conclusions

Wavelet analysis shows great potential for the study of SAPRGs, but it must be used with caution—especially when the amplitude is small, and the seasonal gaps are severe; the gaps can enhance the apparent reality of an alias period. It should be used in combination with light curves, Fourier analysis, and autocorrelation analysis. If SAPRGs are truly irregular, wavelet analysis may not provide any more valid information than the other methods.

The long-term changes in the amplitude of EU Del are of interest, and should be examined in more detail, especially if they are correlated with the long-term changes in mean magnitude. The causes of both are still unknown. A preliminary inspection of the time variations of the amplitude and the mean magnitude shows no obvious correlations.

We plan to apply wavelet analysis to other SAPRGs, especially those for which we have both AAVSO Photoelectric Photometry and robotic telescope data, and other stars with minimal seasonal gaps. The AAVSO Photoelectric Photometry measurements conveniently fill the gaps which occur in the robotic telescope data during the Arizona monsoon season. For this and other reasons, the two datasets are complementary.

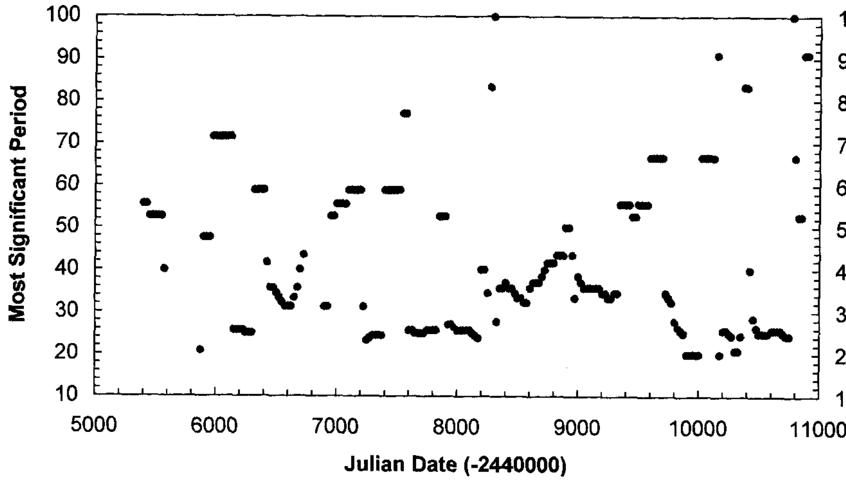

Figure 3. The most significant period of W Boo, as a function of time, based on wavelet analysis of AAVSO Photoelectric Photometry data. A histogram of these results shows peaks at 25, 35±3, and 55±3 days—all of these consistent with previous results using Fourier analysis (Percy and Desjardins 1996).

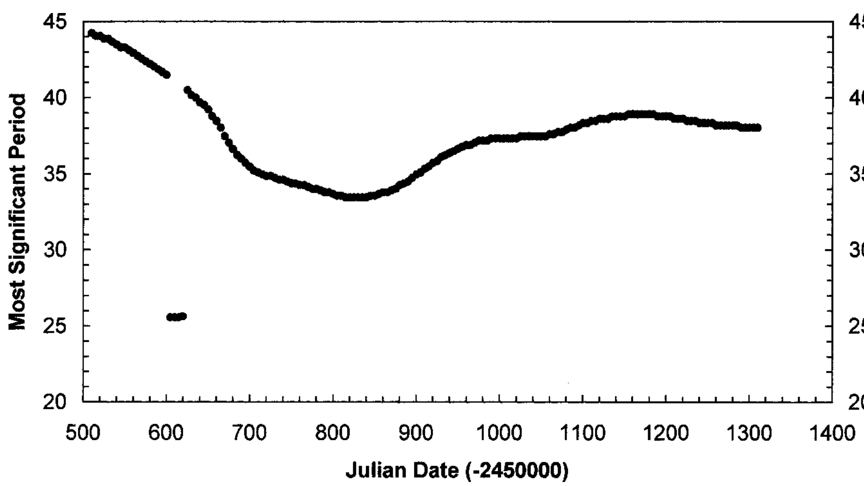

Figure 4. The most significant period of SX UMi, as a function of time, based on wavelet analysis of CCD photometry provided by Josep M. Gomez Forrellad and Enrique Garcia Melendo. A period of about 38 days is present; this value is consistent with the results of autocorrelation analysis. The apparent "wandering" of the period, with time, is probably due to multiperiodicity or irregularity, as explained in the text.

8. Acknowledgements

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