THE S_{10} SYSTEM APPLIED TO CELESTIAL NON-POINT-SOURCE OBJECTS IN DETERMINING SURFACE BRIGHTNESS

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Presented at the 89th Annual Meeting of the AAVSO, October 28, 2000, as "What Magnitude is That Galaxy?"

Abstract

The magnitude system that we use to rate the brightness of stars has major problems when applied to extended objects such as galaxies, nebulae, or comets. The S_{10} brightness system gives a more realistic measure of an object's visibility in the telescope.

1. Introduction

Which is brighter, a 5.7 or 8.4 magnitude galaxy? The answer seems straightforward. Simply plug the magnitudes into equation 1 and solve for I_1/I_2, the ratio of the rate of photon arrival.

$$m_1 - m_2 = -2.50 \, \mathrm{Log}_{10}(I_1/I_2) \tag{1}$$

The result is $I_1/I_2 = 12.0$. This means that the 5.7 magnitude galaxy is twelve times brighter than the 8.7 magnitude galaxy. Not really!

The problem is one of definitions. Equation 1 relates magnitudes to the flux from an object in units of erg/sec/cm^2. Brightness has a slightly different definition. Brightness for extended objects has units of erg/sec/cm^2/steradian. The important difference is that brightness is flux per unit solid angle. This difference takes into account how spread out across the sky the flux from the object appears. If one defocuses a bright star in the telescope, the out-of-focus disk becomes dimmer the farther from focus one goes. The flux is being distributed into an increasingly larger solid angle.

Defocusing a star to match the appearance of a comet is the method devised by Sidgwick (Bishop 1999a) for estimating a comet's magnitude. Stars of known magnitude are defocused until the out-of-focus images are the same size as that of the comet. The magnitude of the comet is then estimated by interpolation from the star magnitudes.

In the example given above with galaxies of 5.7 and 8.4 magnitude we now introduce additional information. The 5.7 magnitude galaxy has apparent size of 67 x 42 arcmin and the 8.4 magnitude galaxy 12 x 5.6 arcmin.

The area of an ellipse, A, is given in equation 2 with d_a and d_b as the major and minor axes. An ellipse approximates the projected shape of a spiral galaxy.

$$A = \frac{\pi}{4} d_a d_b \tag{2}$$

The galaxies in this example are M33 at 5.7 and M 82 at 8.4. M33 has 41 times the area as M82 but M33 has only 12 times the flux. Dividing the flux ratio by the area ratio to get flux per unit area gives the result that M33 has 0.3 times the surface brightness of M82, or that M82 has 3.2 times the surface brightness of M33, and by the definition of brightness, M82 is the brighter of the two.

2. S_{10} system for extended objects

The S_{10} system was devised for quantifying the surface brightness of extended objects. In the S_{10} system the surface brightness of an extended object is given in units of the number of stars of magnitude 10 per square degree that would give the equivalent surface brightness as the object.

Equation 3 gives the formula for calculating the number of tenth magnitude stars, I_g, that would equal the total flux from a galaxy of magnitude, m_g.

$$I_g = \mathrm{Log}_{10}^{-1}(-0.4(m_g-10)) \qquad (3)$$

Equations 4 and 5 give the formula that introduces the area of the object.

$$\frac{I_g}{\frac{\pi}{4} d_a d_b} = \frac{S_{10}}{1 \deg^2} \qquad (4)$$

$$\frac{I_g}{\frac{\pi}{4} d_a d_b} = \frac{S_{10}}{3600 \text{ arcmin}^2} \qquad (5)$$

Equation 4 uses d_a and d_b in degrees while equation 5 uses d_a and d_b in arcmin.

Let us see what are the S_{10} values for M33 and M82.

	m_v	d_a d_b	S_{10}
M33	5.7	(67' x 42')	84
M82	8.4	(12' x 5.6')	286
M1	8.4	(6' x 4')	834

Note that M1 has been added to the list. This supernova remnant, the Crab Nebula, is an extended object and can be treated in the same manner as a galaxy. It has the same magnitude as M82 but is more compact. Its S_{10} value is 2.9 times that of M82, which means that M1 should be easier to spot against a light polluted sky than M82.

The sky has a surface brightness of its own even in an ideal dark observing site. The sky brightness is composed of light from faint stars and natural sky glow apart from local man-made lighting. Average S_{10} values for the night sky in the visual region of the spectrum are 400 at the zenith and 700 at fifteen degrees altitude (Allen 1955a). The moon being above the horizon would increase these values.

The S_{10} values of zodiacal light will give a more intuitive meaning to the S_{10} system (Allen 1955b):

Degrees from Sun	10	40	90	110	150	180
S_{10}	20000	900	210	150	140	200

Note that the zodiacal glow increases when opposite the sun. This increase is caused by the Gegenschein, or counterglow. It is fainter than the natural sky brightness and can only be seen when it is high in the sky and the Milky Way is not in the background.

3. Discussion

The S_{10} system provides more useful information to the observer regarding the visibility of an object than the magnitude alone. One rarely sees the S_{10} system mentioned in astronomy books or popular astronomy magazines. The *Observer's Handbook 2000* (Bishop 1999b) does make a step in the right direction by listing a value of S in units of magnitude per square arcsecond for Galactic Nebulae. These values of surface brightness could be converted to S_{10} values without difficulty, although they are useful in themselves for comparing the visibility of objects in the list.

Even the S_{10} system is inadequate to describe the brightness of a major comet. When the comet is a small featureless blob, the S_{10} is fine. When the comet develops a long tail with knots and streamers, each part will have a different S_{10} value. The magnitude or total flux from such an object is difficult to estimate. Remember the speculations about what magnitude Comet Halley would reach or what was the "real" magnitude of Comet Hale-Bopp. Estimates of the actual magnitude differing by 200 percent were common.

Some observers might protest the S_{10} values of M82 *vs* M1 based on their personal experience at viewing these objects and say that M1 has less surface brightness than M82. The difference may arise from the source of the magnitudes used to calculate the S_{10} values for each object. The magnitudes listed in various Messier Object lists differ from list to list. The source of or method of determining the magnitudes is not documented. These magnitudes may be eye estimates from over one hundred years in the past.

A useful project for someone with a telescope set up for aperture photometry would be to measure the flux of the extended Messier objects through a photometer aperture of known area and calculate an S_{10} value for them. From the known dimensions of the objects a flux in magnitudes could be calculated for comparison with the published magnitudes. Again, there is the problem of objects such as galaxies with brighter nuclear areas that would have nonuniform S_{10} across the object. Still, any move to provide a surface brightness listing would be better than listing the magnitudes alone.

4. Conclusion

The S_{10} system for extended objects is a well-defined and useful system. It should be used more often in observing reports and object lists since it conveys more information relative to the visibility of an object.

References

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