# VX Hydrae: Observation of a Sudden Change in a Pulsating Star 

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Abstract Time-series of VX Hya from March 2005 to April 2010 are analyzed. A period jump is discovered to happen in 2008 in all the pulsation modes, being at its strongest for the overtone.

## 1. Introduction

The $\delta$ Scuti star VX Hydrae was studied by Fitch (1966), who found two main pulsations with periods 0.223 day (fundamental) and 0.173 day (overtone), and up to eighteen harmonics and beats. It was also observed by Templeton et al. (2009) (hereafter T09), who measured the two main pulsations and up to twenty-three harmonics and beats.

The fundamental pulsation is designated as $(1,0)$ and the overtone as $(0,1)$ with the frequencies $n_{10}$ and $n_{01}$, respectively. The other pulsations are designated as (i,j) with the frequencies $n_{\mathrm{ij}}$. They are harmonics/beats of the two main pulsations with $\mathrm{n}_{\mathrm{ij}}=\mathrm{i} \times \mathrm{n}_{10}+\mathrm{j} \times \mathrm{n}_{01}$.

## 2. Discovery of the pulsation change

VX Hya was observed by one of us (MB) with a 203 mm Schmidt-Cassegrain telescope, a Johnson $V$ filter and a SBIG ST7E camera (KAF401E CCD). The exposure durations were 200 seconds (a few measurements have 60s). For the differential photometry, the comparison star was TYC 5482-01347/1 with $V=11.580$ (computed from the Tycho magnitudes owing to Mamajek et al. 2002, 2006).

The observations were carried out from 2005 to 2010, with 1,495 measurements
in thirty-two sessions, and are reported in Table 2. Examples of light curves are in Figures 1-4. All the data are in the AAVSO International Database, observer code BZU (along with $B$ measurements, not used in this paper).

Following Fitch (1966), the observations are fitted with the function of the time t :

$$
\begin{equation*}
\mathrm{V}(\mathrm{t})=\mathrm{V}_{\mathrm{m}}-2.5 \log \left[1+\sum_{\mathrm{i}, \mathrm{j}}\left(\mathrm{a}_{\mathrm{ij}} \sin \left(2 \pi \mathrm{n}_{\mathrm{ij}} \mathrm{t}\right)+\mathrm{b}_{\mathrm{ij}} \cos \left(2 \pi \mathrm{n}_{\mathrm{ij}} \mathrm{t}\right)\right)\right] \tag{1}
\end{equation*}
$$

where $V_{m}$ is the non-pulsed part of the brightness and the sum is carried out over the ( $\mathrm{i}, \mathrm{j}$ ) pulsations with frequencies $\mathrm{n}_{\mathrm{ij}}$.

It is not possible to obtain a good single set of parameters for all the seasons. A good set was obtained for the March 2005-December 2006 observations (see Figures 1 and 2), showing that the pulsations change very little during that period. But this set of parameters does not fit the February 2008-January 2010 data; the observations are late when compared with function (1), and become later and later (see Figures 3 and 4). A change in the pulsations happened between December 2006 and February 2008, making the frequencies smaller (or the periods longer). Up to six pulsations were used for the fit, but the period change is already visible if one uses only the two main pulsations.

## 3. Analysis of the pulsation change

The observations were fitted with function (1), season by season, the following way:

- the two main pulsations were used: the fundamental $(1,0)$ and the overtone $(0,1)$, two harmonics $(2,0)$ and $(0,2)$, and two beats $(1,1)$ and $(-1,1)$. These six pulsations are those with the greatest amplitudes. There are other pulsations (e.g. T09 detected a total of twenty-five pulsations), but they have smaller amplitudes and are neglected in this study. (Actually an analysis with only the two main pulsations leads to results that are very similar to those shown here);
- for $\mathrm{V}_{\mathrm{m}}$ the average magnitude over the season (as Fitch 1966) was used. This approximation is valid because the duration of the observations for a season is much longer than the pulsation periods. Actually, it will be checked that $\mathrm{V}_{\mathrm{m}}$ calculated that way is constant over the seasons;
- the $a_{\mathrm{ij}}$ and $\mathrm{b}_{\mathrm{ij}}$ coefficients are then determined by leastsquares (calculating $12 \times 12$ matrixes and inverting them).

Additional observations from the AAVSO International Database were also used, so as to have more data and data that are independently acquired. They were obtained with telescope apertures of 200 to 250 mm , SBIG ST7 and ST9
cameras, and the same filter and comparison star as above. The data are from the co-authors SD (observer code DKS), RP (PRX), and GS (SAH), with a total of 10,523 measurements in 60 sessions, and are reported in Table 2 (some of these data were also used by T09). These observations were also fitted the same way as explained above.

The phase of each pulsation is computed by:

$$
\begin{equation*}
\phi_{\mathrm{ij}}=\operatorname{atan} 2\left(\mathrm{a}_{\mathrm{ij}}, \mathrm{~b}_{\mathrm{ij}}\right) \tag{2}
\end{equation*}
$$

The frequencies $n_{i j}$ are then adjusted so that the phases $\phi_{\mathrm{ij}}$ vary little over the 2004-2007 seasons. The adopted frequencies are in Table 1. The difference with the frequencies of T09 for 2006 is smaller than $10^{-4}$ for $\mathrm{n}_{10}$ and $\mathrm{n}_{01}$ (actually within their uncertainty for $\mathrm{n}_{10}$ in 2006), and smaller than $5 \times 10^{-4}$ for the four harmonic/beat frequencies. The difference with the frequencies for 1955-1959 of Fitch (1966) is smaller than $2 \times 10^{-4}$ for $n_{10}$ and $n_{01}$. One can check that the four frequencies $n_{11} \ldots n_{02}$ are harmonics/beats of $n_{10}$ and $n_{01}$ with a precision better than $10^{-4}$, for example, $\mathrm{n}_{11}=\mathrm{n}_{10}+\mathrm{n}_{01}$.

The amplitude of each pulsation is computed by:

$$
\begin{equation*}
A_{i j}=\sqrt{a_{i j}^{2}+b_{i j}^{2}} \tag{3}
\end{equation*}
$$

The resulting parameters $\phi_{\mathrm{ij}}$ and $\mathrm{A}_{\mathrm{ij}}$ for each data set are listed in Table 2. The uncertainties in the dates (HJD) of the data sets are the standard deviations of the dates of the measurements. To evaluate the uncertainties on the phases and the amplitudes, $\mathrm{V}_{\mathrm{m}}$ is varied by $\pm 0.01$ magnitude and one observes how $\phi_{\mathrm{ij}}$ and $\mathrm{A}_{\mathrm{ij}}$ change. The uncertainties are found to be the largest for the $(-1,1)$ beat. This is because this pulsation has the longest period ( $\mathrm{P}_{-11}=0.76$ day) and the corresponding parameters are then less constrained by the finite data sets.

## 4. Results

Figure 5 shows the phases as a function of time. There is clearly a sudden phase shift in 2008 . The $(1,0)$ pulsation has its phase modified by about $20^{\circ}$, the $(0,1)$ pulsation by $60^{\circ}$, and the phases of the other pulsations change, too. This corresponds to decreases in the pulsation frequencies (and increases in the periods).

A phase shift over time is equivalent to a frequency change. With the time scale of the phase shift being much longer than the pulsation periods, the change $\Delta n_{i j}$ of the frequency $n_{i j}$ can be computed from the phase shifts $\Delta \phi_{i j}$ as:

$$
\begin{equation*}
\Delta \mathrm{n}_{\mathrm{ij}}=\frac{\Delta \phi_{\mathrm{ij}}}{2 \pi \Delta \mathrm{t}} \tag{4}
\end{equation*}
$$

where $\Delta t$ is the time interval. The variations of the frequencies, taking the 2005-2006 data set (of observer DKS) as an origin, are computed for the (1,0) and the $(0,1)$ pulsations and the results are shown Table 3 and Figure 6:

- for the pulsation $(1,0)$ there is a jump in the frequency between THJD 4100 and 4500 , then the frequency stays constant after a decrease of $-(46.7 \pm 2.7) \times 10^{-6}$ day $^{-1}$ (the uncertainty is the standard deviation over 2008-2010);
- for the pulsation $(0,1)$ the frequency jump lasts from THJD 4100 to 4800 with a decrease of at least $-100 \times 10^{-6}$ day $^{-1}$. It appears that it stays constant thereafter (more observations would be needed here).

That is, the jump for the $(0,1)$ overtone is twice as strong as the jump for the $(1,0)$ fundamental and lasts twice longer.

The other four pulsations are harmonics or beats of the first two. Their phase variations may then be computed from the phase variations of the fundamental and of the overtone, and compared with the observed phase variations. There should be no difference. For example, for the $(1,1)$ pulsation, the computed phase variation is $\Delta \phi_{11}{ }^{\text {comput }}=\Delta \phi_{10}+\Delta \phi_{01}$ and one can check that it is roughly equal to the observed $\Delta \phi_{11}$. The residuals between the observed and computed values are shown Table 3 and Figure 7. They are roughly around 0, the worst being the $(-1,1)$ pulsation, but this one has large error bars, as explained above.

The variation of the amplitudes $\mathrm{A}_{\mathrm{ij}}$ of the oscillations versus time may be studied, although there is a need for caution because the data come from different observers (with different telescope setups, and so on). This is shown in Figure 8: there is a trend for an increase of the amplitudes before the 2008 period change followed by a decrease. Again, the effect is stronger for the $(0,1)$ overtone (and its $(0,2)$ harmonic) than for the $(1,0)$ fundamental.

For the non-pulsed part of the brightness $\left(\mathrm{V}_{\mathrm{m}}\right)$, from author MB (observer code BZU) observations only (so as to have homogenous data sets taken with the same setup) a significant variation cannot be detected (standard deviation is 0.014 mag ) in 2004-2005 and 2006-2010. There is no variation either (standard deviation is 0.008 ) from author SD (observer DKS) observations in 2005-2006 and 2008-1010. This both allows checking the validity of the approximation of using an average for the determination of $\mathrm{V}_{\mathrm{m}}$ and gives an upper limit on variation on $V_{m}$ coming from the star.

## 5. Discussion

These measurements of amplitudes and phases of the oscillations may be compared with those ofT09, especially for the 2005-2006 season. The amplitudes $\mathrm{M}_{\mathrm{ij}}$ of T09 in magnitude are connected to the $\mathrm{A}_{\mathrm{ij}}$ through:

$$
\begin{equation*}
M_{i j}=\frac{2.5}{\log (10)}\left(A_{i j}-\frac{A_{i j}{ }^{2}}{2}+\frac{A_{i j}{ }^{3}}{3}-\ldots\right) \tag{5}
\end{equation*}
$$

T 09 lists the magnitudes of the oscillations at $\mathrm{t}_{0,2006}=2453807.3515$. The evaluation of the amplitudes agrees with theirs within 3 mmag for the two main tones, and within 7 mmag for the four harmonics and beats.

T09 also lists the phases of the oscillations at $t_{0,2006}$. These phases can be recalculated as $2 \pi n_{i \mathrm{ij}} \mathrm{t}_{0,2006}+\phi_{\mathrm{ij}}$. The agreement is within $2^{\circ}$ for the two main tones and within $9^{\circ}$ for the four harmonics and beats (modulo $270^{\circ}$, which comes from different models of the oscillations).

The agreement with T09 may be considered as satisfying despite the small differences, especially in view of the small number of harmonics/beats used here.

The increase in amplitude for the overtone $(0,1)$ before 2008 was also detected by T09. The increase for the fundamental $(1,0)$ was not seen by T09 but here the observations are over a longer time and the change is smaller.

The analysis presented here, which is somewhat like an $\mathrm{O}-\mathrm{C}$ study, is very sensitive to the difference between two data sets. It is more sensitive than, for example, determining the absolute frequency of each data set and taking the difference to search for a variation.

VXHya is an easy to observe $\delta$ Sct star with only one large amplitude overtone. A sudden period jump was discovered that is stronger and lasts longer for the overtone than for the fundamental and that appears to propagate to the harmonics and the beats. Also, the amplitudes of the pulsations got stronger before the period jump. There is a need for a physical explanation of the underlying process and such observations should be helpful to understand how the different tones are coupled.

## 6. Acknowledgement

The use of the AAVSO International Database and the contributions of data from the co-authors for this amateur research are acknowledged.

## References

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Templeton, M. R., Samolyk, G., Dvorak, S., Poklar, R., Butterworth, N., and Gerner, H. 2009, Publ. Astron. Soc. Pacific, 121, 1076.

Table 1. The adopted frequencies (in day ${ }^{-1}$ ), after adjustment to minimize the phase shifts for the 2004-2007 seasons (around HJD 2453800).

| harmonic | frequency $\left(d^{-1}\right)$ |
| :---: | :---: |
| $\mathrm{n}_{10}$ | 4.476493 |
| $\mathrm{n}_{01}$ | 5.789900 |
| $\mathrm{n}_{11}$ | 10.266397 |
| $\mathrm{n}_{20}$ | 8.952962 |
|  | $\mathrm{n}_{-11}$ |
|  | 1.313422 |
| $\mathrm{n}_{02}$ | 11.579792 |

Table 2. The parameters that fit the different sets of observations of VX Hya ( $\phi$ the phases in ${ }^{\circ}$, A the amplitudes).

| Season <br> Observer | $\left\lvert\, \begin{gathered} 2004- \\ 2005 \\ \text { BZU } \end{gathered}\right.$ | $\begin{gathered} 2005- \\ 2006 \\ \text { DKS } \end{gathered}$ | SAH | BZU | $6-2007$ <br> PRX | SAH | $\begin{gathered} 2007-2008 \\ \\ \text { BZU } \\ \\ \\ \text { PRX+ } \\ \text { DKS } \end{gathered}$ |  | $200$ <br> DKS | $8-2009$ <br> BZU | $\begin{array}{r} 20 \\ \text { BZU } \end{array}$ | 9-2010 <br> DKS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of sessions | 6 | 16 | 5 | 4 | 11 | 5 | 7 | 4 | 3 | 9 | 6 | 16 |
| Duration (h) | 31.8 | 111.7 | 27.1 | 17.4 | 46.8 | 22.2 | 48.2 | 16.3 | 17.6 | 36.9 | 27.7 | 64.5 |
| No. of obs. | 248 | 3098 | 753 | 162 | 1203 | 626 | 346 | 429 | 712 | 354 | 385 | 3702 |
| HJD-2450000 | $\begin{array}{r} 3445.1 \\ \pm 2.8 \end{array}$ | $\begin{array}{r} 3794.1 \\ \pm 18.2 \end{array}$ | $\begin{array}{r} 4096.0 \\ \pm 5.9 \end{array}$ | $\begin{array}{r} 4096.2 \\ \pm 1.0 \end{array}$ | $\begin{array}{r} 4165.3 \\ \pm 15.7 \end{array}$ | $\begin{array}{r} 4194.8 \\ \pm 16.4 \end{array}$ | $\begin{array}{r} 4507.6 \\ \pm 2.0 \end{array}$ | $\begin{array}{r} 4558.6 \\ \pm 7.1 \end{array}$ | $\begin{array}{r} 4853.4 \\ \pm 3.3 \end{array}$ | $\begin{array}{r} 4873.7 \\ \pm 37.1 \end{array}$ | $\begin{array}{r} 5209.6 \\ \pm 34.8 \end{array}$ | $\begin{array}{r} 5280.0 \\ \pm 17.4 \end{array}$ |
| $\phi_{10}\left({ }^{\circ}\right)$ | 122.5 $\pm 0.2$ | 124.5 $\pm 0.4$ | 120.0 $\pm 0.5$ | 123.1 $\pm 0.1$ | 122.6 $\pm 0.7$ | 119.2 $\pm 0.6$ | 114.1 $\pm 0.2$ | $\begin{array}{r} 110.6 \\ \pm 1.8 \end{array}$ | $\begin{array}{r} 105.3 \\ \pm 0.4 \end{array}$ | $\begin{array}{r} 105.9 \\ \pm 0.1 \end{array}$ | $\begin{array}{r} 102.4 \\ \pm 0.9 \end{array}$ | $\begin{array}{r} 100.3 \\ \pm 0.7 \end{array}$ |
| $\phi_{01}$ | 172.2 <br> $\pm 0.2$ | $\begin{array}{r} 173.9 \\ \pm 0.1 \end{array}$ | $\begin{array}{r} 173.5 \\ \pm 1.1 \end{array}$ | $\begin{array}{r} 172.4 \\ \pm 0.1 \end{array}$ | $\begin{array}{r} 172.4 \\ \pm 0.1 \end{array}$ | $\begin{array}{r} 176.9 \\ \pm 0.7 \end{array}$ | $\begin{array}{r} 156.0 \\ \pm 0.1 \end{array}$ | $\begin{array}{r} 154.3 \\ \pm 1.1 \end{array}$ | $\begin{array}{r} 134.5 \\ \pm 1.2 \end{array}$ | 129.0 <br> $\pm 0.4$ | 114.4 $\pm 0.4$ | $\begin{array}{r} 109.5 \\ \pm 0.3 \end{array}$ |
| $\phi_{11}$ | $\begin{array}{r} 312.2 \\ \pm 0.5 \end{array}$ | $\begin{array}{r} 316.5 \\ \pm 0.5 \end{array}$ | $\begin{array}{r} 321.6 \\ \pm 2.6 \end{array}$ | $\begin{array}{r} 316.5 \\ \pm 0.9 \end{array}$ | $\begin{array}{r} 312.1 \\ \pm 0.5 \end{array}$ | $\begin{array}{r} 325.8 \\ \pm 1.1 \end{array}$ | $\begin{array}{r} 289.0 \\ \pm 0.3 \end{array}$ | $\begin{array}{r} 288.9 \\ \pm 2.6 \end{array}$ | $\begin{array}{r} 249.3 \\ \pm 2.0 \end{array}$ | $\begin{array}{r} 247.2 \\ \pm 1.0 \end{array}$ | 224.3 <br> $\pm 0.8$ | $\begin{array}{r} 234.9 \\ \pm 0.8 \end{array}$ |
| $\phi_{20}$ | $\begin{array}{r} 165.8 \\ \pm 1.0 \end{array}$ | 167.3 $\pm 0.8$ | 182.3 $\pm 2.8$ | $\begin{array}{r} 156.0 \\ \pm 0.2 \end{array}$ | $\begin{array}{r} 166.4 \\ \pm 1.3 \end{array}$ | $\begin{array}{r} 160.5 \\ \pm 0.8 \end{array}$ | 153.4 <br> $\pm 1.0$ | $\begin{array}{r} 162.3 \\ \pm 1.8 \end{array}$ | 131.5 $\pm 0.6$ | 135.3 $\pm 3.6$ | 141.2 $\pm 0.3$ | 135.6 $\pm 0.8$ |
| $\phi_{-11}$ | 206.4 $\pm 3.0$ | $\begin{array}{r} 207.3 \\ \pm 1.1 \end{array}$ | $\begin{aligned} & 237.9 \\ & \pm 16.1 \end{aligned}$ | $\begin{array}{r} 206.1 \\ \pm 3.5 \end{array}$ | 205.4 $\pm 2.0$ | $\begin{array}{r} 203.4 \\ \pm 7.3 \end{array}$ | $\begin{array}{r} 196.1 \\ \pm 0.2 \end{array}$ | $\begin{aligned} & 244.1 \\ & \pm 26.4 \end{aligned}$ | $\begin{array}{r} 172.0 \\ \pm 5.8 \end{array}$ | $\begin{array}{r} 162.4 \\ \pm 4.4 \end{array}$ | $\begin{array}{r} 130.5 \\ \pm 7.0 \end{array}$ | $\begin{array}{r} 163.5 \\ \pm 5.5 \end{array}$ |

Table 2. The parameters that fit the different sets of observations of VX Hya ( $\phi$ the phases in ${ }^{\circ}$, A the amplitudes), continued.

| Season Observer | $\begin{gathered} 2004- \\ 2005 \\ \text { BZU } \end{gathered}$ | $\begin{gathered} 2005- \\ 2006 \\ \text { DKS } \end{gathered}$ | SAH | $\begin{array}{r} 20 \\ \text { BZU } \end{array}$ | $\begin{array}{r} 6-2007 \\ \text { PRX } \end{array}$ | SAH | $\begin{array}{r} 200 \\ \text { BZU } \\ \hline \end{array}$ | -2008 <br> PRX+ <br> DKS | $200$ <br> DKS | $\begin{gathered} 8-2009 \\ \text { BZU } \end{gathered}$ | $\begin{array}{r} 20 \\ \text { BZU } \end{array}$ | $9-2010$ <br> DKS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\phi_{02}$ | $\begin{array}{r} 178.7 \\ \pm 0.4 \end{array}$ | $\begin{array}{r} 184.5 \\ \pm 1.3 \end{array}$ | $\begin{array}{r} 184.1 \\ \pm 2.2 \end{array}$ | $\begin{array}{r} 193.8 \\ \pm 0.3 \end{array}$ | $\begin{array}{r} 178.6 \\ \pm 1.0 \end{array}$ | $\begin{array}{r} 152.0 \\ \pm 0.8 \end{array}$ | $\begin{array}{r} 158.7 \\ \pm 0.1 \end{array}$ | $\begin{array}{r} 141.9 \\ \pm 0.3 \end{array}$ | $\begin{array}{r} 104.9 \\ \pm 0.7 \end{array}$ | $\begin{aligned} & 99.1 \\ & \pm 2.1 \end{aligned}$ | $\begin{aligned} & 61.7 \\ & \pm 0.5 \end{aligned}$ | $\begin{aligned} & 76.6 \\ & \pm 1.2 \end{aligned}$ |
| $\mathrm{A}_{10}$ | $\begin{gathered} 0.1280 \\ \pm 0.0020 \pm \end{gathered}$ | $\begin{gathered} 0.1323 \\ \pm 0.0011= \end{gathered}$ | $\begin{array}{r} 0.1318 \\ \pm 0.0001 \end{array}$ | $\begin{array}{r} 0.1378 \\ \pm 0.0001 \end{array}$ | $\begin{aligned} & 0.1452 \\ & 0.0009 \end{aligned}$ | $\begin{array}{r} 0.1626 \\ \pm 0.0019 \end{array}$ | $\begin{array}{r} 0.1368 \\ \pm 0.0012 \end{array}$ | $\begin{array}{r} 0.1323 \\ \pm 0.0059 \end{array}$ | $\begin{array}{r} 0.1405 \\ \pm 0.0022 \end{array}$ | $\begin{array}{r} 0.1263 \\ \pm 0.0001 \end{array}$ | $\begin{array}{r} 0.1258 \\ \pm 0.0004 \end{array}$ | $\begin{aligned} & 0.1277 \\ & 0.0008 \end{aligned}$ |
| $\mathrm{A}_{01}$ | $\begin{gathered} 0.0949 \\ \pm 0.0008 \pm \end{gathered}$ | $\begin{array}{r} 0.1146 \\ \pm 0.0006 \end{array}$ | $\begin{array}{r} 0.1171 \\ \pm 0.0004 \end{array}$ | $\begin{array}{r} 0.1336 \\ +0.0009 \end{array}$ | $\begin{array}{r} 0.1356 \\ \pm 0.0011 \end{array}$ | $\begin{array}{r} 0.1292 \\ \pm 0.0041 \end{array}$ | $\begin{array}{r} 0.1286 \\ \pm 0.0008 \end{array}$ | $\begin{array}{r} 0.1383 \\ +0.0075 \end{array}$ | $\begin{array}{r} 0.1127 \\ \pm 0.0006 \end{array}$ | $\begin{array}{r} 0.1107 \\ \pm 0.0002 \end{array}$ | $\begin{array}{r} 0.1063 \\ \pm 0.0008 \end{array}$ | $\begin{gathered} 0.1126 \\ =0.0002 \end{gathered}$ |
| $\mathrm{A}_{11}$ | $\begin{gathered} 0.0452 \\ \pm 0.0005 \pm \end{gathered}$ | $\begin{array}{r} 0.0556 \\ \pm 0.0004 \end{array}$ | $\begin{array}{r} 0.0625 \\ \pm 0.0003 \end{array}$ | $\begin{array}{r} 0.0659 \\ +0.0005 \end{array}$ | $\begin{array}{r} 0.0744 \\ \pm 0.0003 \end{array}$ | $\begin{array}{r} 0.0482 \\ \pm 0.0004 \end{array}$ | $\begin{array}{r} 0.0659 \\ \pm 0.0003 \end{array}$ | $\begin{array}{r} 0.0642 \\ \pm 0.0026 \end{array}$ | $\begin{array}{r} 0.0593 \\ \pm 0.0009 \end{array}$ | $\begin{array}{r} 0.0600 \\ \pm 0.0040 \end{array}$ | $\begin{array}{r} 0.0526 \\ \pm 0.0004 \end{array}$ | $\begin{array}{r} 0.0549 \\ =0.0002 \end{array}$ |
| $\mathrm{A}_{20}$ | $\begin{gathered} 0.0392 \\ \pm 0.0005 \pm \end{gathered}$ | $\begin{array}{r} 0.0375 \\ \pm 0.0004 \end{array}$ | $\begin{array}{r} 0.0306 \\ \pm 0.0016 \end{array}$ | $\begin{array}{r} 0.0523 \\ \pm 0.0022 \end{array}$ | $\begin{array}{r} 0.0467 \\ \pm 0.0012 \end{array}$ | $\begin{array}{r} 0.0641 \\ \pm 0.0007 \end{array}$ | $\begin{array}{r} 0.0368 \\ \pm 0.0006 \end{array}$ | $\begin{array}{r} 0.0453 \\ \pm 0.0035 \end{array}$ | $\begin{array}{r} 0.0434 \\ \pm 0.0009 \end{array}$ | $\begin{array}{r} 0.0404 \\ \pm 0.0029 \end{array}$ | $\begin{array}{r} 0.0351 \\ \pm 0.0003 \end{array}$ | $\begin{array}{r} 0.0365 \\ =0.0012 \end{array}$ |
| $\mathrm{A}_{-11}$ | $\begin{gathered} 0.0358 \\ \pm 0.0032 \pm \end{gathered}$ | $\begin{array}{r} 0.0372 \\ \pm 0.0013= \end{array}$ | $\begin{array}{r} 0.0248 \\ \pm 0.0030 \end{array}$ | $\begin{array}{r} 0.0529 \\ \pm 0.0009 \end{array}$ | $\begin{array}{r} 0.0557 \\ \pm 0.0011 \end{array}$ | $\begin{array}{r} 0.0524 \\ \pm 0.0102 \end{array}$ | $\begin{array}{r} 0.0450 \\ \pm 0.0004 \end{array}$ | $\begin{array}{r} 0.0290 \\ \pm 0.0042 \end{array}$ | $\begin{array}{r} 0.0535 \\ \pm 0.0013 \end{array}$ | $\begin{array}{r} 0.0359 \\ \pm 0.0031 \end{array}$ | $\begin{array}{r} 0.0511 \\ \pm 0.0012 \end{array}$ | $\begin{array}{r} 0.0335 \\ =0.0037 \end{array}$ |
| $\mathrm{A}_{02}$ | $\begin{gathered} 0.0248 \\ \pm 0.0008 \pm \end{gathered}$ | $\begin{array}{r} 0.0252 \\ \pm 0.0001 \end{array}$ | $\begin{array}{r} 0.0422 \\ \pm 0.0018 \end{array}$ | $\begin{array}{r} 0.0333 \\ \pm 0.0007 \end{array}$ | $\begin{array}{r} 0.0404 \\ \pm 0.0010 \end{array}$ | $\begin{array}{r} 0.0392 \\ \pm 0.0010 \end{array}$ | $\begin{array}{r} 0.0341 \\ \pm 0.0006 \end{array}$ | $\begin{array}{r} 0.0522 \\ +0.0008 \end{array}$ | $\begin{array}{r} 0.0284 \\ \pm 0.0024 \end{array}$ | $\begin{array}{r} 0.0220 \\ \pm 0.0007 \end{array}$ | $\begin{array}{r} 0.0275 \\ \pm 0.0008 \end{array}$ | $\begin{array}{r} 0.0299 \\ =0.0001 \end{array}$ |

Table 3. The frequency variations of the fundamental $(1,0)$ and of the overtone $(0,1)$ of VX Hya from the 2005-2006 season (in $10^{-6} \mathrm{~d}^{-1}$ ). The phase residuals between the observed phase variations and the one computed from the frequency variations of $(1,0)$ and $(0,1)$, again taking the 2005-2006 season as the origin (in ${ }^{\circ}$ ).

| Season <br> Observer | $\begin{gathered} 2004 \\ 2005 \\ \text { BZU } \end{gathered}$ | $\begin{gathered} 2005- \\ 2006 \\ \text { DKS } \end{gathered}$ | 2006-2007 |  |  |  | 2007-2008 |  | 2008-2009 |  | 2009-2010 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | SAH | BZU | PRX | SAH | BZU | PRX+ DKS | DKS | BZU | BZU | DKS |
| HJD-2450000 | $\begin{array}{r} 3445.1 \\ \pm 2.8 \end{array}$ | $\begin{array}{r} 3794.1 \\ \pm 18.2 \end{array}$ | $\begin{array}{r} 4096.0 \\ \pm 5.9 \end{array}$ | $\begin{array}{r} 4096.2 \\ \pm 1.0 \end{array}$ | $\begin{array}{r} 4165.3 \\ \pm 15.7 \end{array}$ | $\begin{array}{r} 4194.8 \\ \pm 16.4 \end{array}$ | $\begin{array}{r} 4507.6 \\ \pm 2.0 \end{array}$ | $\begin{array}{r} 4558.6 \\ \pm 7.1 \end{array}$ | $\begin{array}{r} 4853.4 \\ \pm 3.3 \end{array}$ | $\begin{array}{r} 4873.7 \\ \pm 37.1 \end{array}$ | $\begin{array}{r} 5209.6 \\ \pm 34.8 \end{array}$ | $\begin{array}{r} 5280.0 \\ \pm 17.4 \end{array}$ |
| $\Delta \mathrm{n}_{10}\left(10^{-6} \mathrm{~d}^{-1}\right)$ | $\begin{array}{r} -15.9 \\ \pm 5.2 \end{array}$ | 0.0 | $\begin{array}{r} -41.4 \\ \pm 10.9 \end{array}$ | $\begin{array}{r} -12.9 \\ \pm 4.5 \end{array}$ | $\begin{array}{r} -14.2 \\ \pm 8.8 \end{array}$ | $\begin{array}{r} -36.7 \\ \pm 9.2 \end{array}$ | $\begin{array}{r} -40.5 \\ \pm 3.2 \end{array}$ | $\begin{array}{r} -50.5 \\ \pm 9.3 \end{array}$ | $\begin{array}{r} -50.4 \\ \pm 2.8 \end{array}$ | $\begin{array}{r} -47.9 \\ \pm 3.0 \end{array}$ | $\begin{array}{r} -43.4 \\ \pm 3.8 \end{array}$ | $\begin{array}{r} -45.2 \\ \pm 2.7 \end{array}$ |
| $\Delta \mathrm{n}_{01}$ | $\begin{array}{r} -13.5 \\ \pm 3.1 \end{array}$ | 0.0 | $\begin{array}{r} -3.7 \\ \pm 11.3 \end{array}$ | $\begin{array}{r} -13.8 \\ \pm 2.2 \end{array}$ | $\begin{array}{r} -11.2 \\ \pm 1.5 \end{array}$ | $\begin{gathered} 20.8 \\ \pm 6.8 \end{gathered}$ | $\begin{array}{r} -69.7 \\ \pm 2.6 \end{array}$ | $\begin{array}{r} -71.2 \\ \pm 6.2 \end{array}$ | $\begin{array}{r} -103.3 \\ \pm 5.1 \end{array}$ | $\begin{array}{r} -115.5 \\ \pm 5.7 \end{array}$ | $\begin{array}{r} -116.8 \\ \pm 4.1 \end{array}$ | $\begin{array}{r} -120.4 \\ \pm 2.7 \end{array}$ |
| $\begin{aligned} & \Delta \phi_{11}-\left(^{\circ}\right) \\ & \Delta \phi_{11} \text { comput } \end{aligned}$ | -0.6 $\pm 1.9$ | 0.0 $\pm 2.0$ | 10.0 $\pm 5.2$ | 2.9 $\pm 1.9$ | -1.0 $\pm 2.1$ | 11.6 $\pm 3.4$ | 0.8 $\pm 1.5$ | 5.9 $\pm 6.5$ | -8.6 $\pm 4.5$ | -5.8 $\pm 2.5$ | -10.6 $\pm 3.1$ | 7.0 $\pm 2.7$ |
| $\begin{aligned} & \Delta \phi_{20}{ }^{-} \\ & \Delta \phi_{20}{ }^{\text {comput }} \end{aligned}$ | 2.5 $\pm 2.9$ | 0.0 $\pm 3.2$ | 24.0 $\pm 5.4$ | -8.5 $\pm 1.8$ | 2.9 $\pm 4.2$ | 3.8 $\pm 3.6$ | 6.9 $\pm 2.9$ | 22.8 $\pm 7.0$ | 2.6 $\pm 2.8$ | 5.2 $\pm 5.3$ | 18.1 $\pm 3.7$ | 16.7 $\pm 3.7$ |
| $\begin{aligned} & \Delta \phi_{-11}- \\ & \Delta \phi_{-11} \text { comput } \end{aligned}$ | -1.2 $\pm 4.9$ | 0.0 $\pm 3.1$ | 26.5 $\pm 19.3$ | -1.1 $\pm 5.1$ | -2.3 $\pm 4.2$ | -12.2 $\pm 10.1$ | -3.7 $\pm 2.0$ | 42.5 $\pm 30.9$ | -15.1 $\pm 8.8$ | -18.6 $\pm 6.4$ | -39.4 $\pm 9.8$ | -3.6 $\pm 7.9$ |
| $\begin{aligned} & \Delta \phi_{02}- \\ & \Delta \phi_{02}^{\text {comput }} \end{aligned}$ | -2.4 $\pm 2.2$ | 0.0 $\pm 2.9$ | 0.4 $\pm 5.8$ | 12.3 $\pm 1.8$ | -2.9 $\pm 2.4$ | -38.5 $\pm 3.7$ | 10.0 $\pm 1.6$ | $\begin{aligned} & -3.4 \\ & \pm 4.0 \end{aligned}$ | -0.8 $\pm 4.4$ | 4.4 $\pm 4.3$ | -3.8 $\pm 2.6$ | 20.9 $\pm 3.1$ |



Figure 1. VX Hya light curve for 16 March 2005. The dots are the observations; the line is the model with six pulsations that fits the 2005-2006 observations.


Figure 2. VX Hya light curve for 27 December 2006. The dots are the observations; the line is the model with six pulsations that fits the 2005-2006 observations.


Figure 3. VX Hya light curve for 12 February 2008. The observations are 11 minutes late from the model, that is, the pulsation frequencies decreased.


Figure 4. VX Hya light curve for 11 January 2010. The observations are 15 minutes late from the $(1,0)$ pulsation and 40 mn from the $(0,1)$ pulsation of the model.


Figure 5. VX Hya phases of the pulsations versus time. From top to bottom: $(1,1)$ pulsation (dashed line); ( $-1,1$ ) pulsation (dashed line, with large error bars); $(0,2)$ pulsation (dotted line); ( 0,1 ) pulsation (full line); ( 2,0 ) pulsation (dotted line); ( 1,0 ) pulsation (full line).


Figure 6. VX Hya frequency variations for the ( 1,0 ) fundamental (square symbols) and for the ( 0,1 ) overtone (diamond symbols), taking the 2005-2006 observations $(\mathrm{HJD}=2,453,800)$ as 0 .


Figure 7.VX Hya. For the harmonics and beats, the residuals between the observed phase variations and the ones computed from the variations of the fundamental and of the overtone, taking the 2005-2006 observations (HJD $=2,453,800$ ) as 0 .


Figure 8. VX Hya amplitudes of the pulsations versus time. From top to bottom: $(1,0)$ pulsation (full line); ( 0,1 ) pulsation (full line); ( 1,1 ) pulsation (dashed line); $(2,0)$ pulsation (dashed line); ( $-1,1$ ) pulsation (dashed line, with large error bars); $(0,2)$ pulsation (dotted line).

