Period Changes in RV Tauri and SRd Variables

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Abstract RV Tauri (RVT) stars and semiregular pulsating variable supergiants of spectral types F, G, and K (SRd variables in the *General Catalogue of Variable Stars*) are undergoing "blue loops" from the asymptotic giant branch (AGB) in the Hertzsprung-Russell diagram, or are in transition from the AGB to the white dwarf stage. They should therefore show period changes due to their evolution. We have studied five such stars—AG Aur, AV Cyg, SX Her, UZ Oph, and TX Per—using up to a century of data, including data from the AAVSO International Database. We show (O–C) diagrams for these stars, fit parabolae to these, and calculate the characteristic time scales of period change. These five stars also, however, show strong evidence of random cycle-to-cycle period fluctuations, such as have been found in other stars of these types. These complicate the interpretation of the (O–C) diagrams in terms of evolutionary period changes, but the time scales that we derive are not inconsistent with the expected evolutionary time scales.

1. Introduction

RV Tauri (RVT) variables are old, low-mass, pulsating yellow supergiants whose light curves are characterized by alternating deep and shallow minima. Some RVT variables also show long secondary periods; those that do are classified as RVB; those that do not are classified as RVA. Yellow semiregular (SRd) variables are old, low-mass, yellow supergiants whose pulsation is semiregular at best.

It has long been suspected that RVT and SRd variables are related to each other, and to the Population II Cepheid (CW) variables, which are old, low-mass, pulsating yellow supergiants with regular variability. Some CW variables show incipient RVT behavior, or are slightly irregular. In most RVT variables, the behavior is semiregular in the sense that it deviates from the "alternating deep and shallow minima" to a greater or lesser extent. The distinction between CW, RVT, and SRd is often based on observation of a limited number of cycles; the classification might be different if a larger number of cycles had been observed and carefully studied. Percy *et al.* (2003), and Percy and Mohammed (2004) have recently used *self-correlation analysis* to characterize the light curves of RVT stars. They find that some RVT stars show only a marginal tendency (if any) to alternate between deep and shallow minima, and that some SRd variables show a high degree of periodicity. Alcock *et al.* (1998) established a link between the period-luminosity relations for CW and RVT stars. Studies of the physical properties of RVT and SRd variables (e.g., Dawson 1979; Wahlgren 1993) suggest that the differences *between* the classes are smaller than the variations of the properties of the stars *within* them.

In terms of evolutionary status, CW, RVT, and SRd variables are often lumped together. They are sun-like stars near the end of their lifetimes. They are either undergoing "blue loops" from the asymptotic-giant branch (AGB) to the yellow supergiant region in the Hertzsprung-Russell Diagram (HRD), due to thermal instabilities or "flashes" in their hydrogen- and helium-burning shells or, in the case of the most luminous stars, are in transition from the AGB to the white dwarf region (Gingold 1976). The time during which a star is in the instability strip during a blue loop is a complex function of the properties of the star, and of the physical assumptions made, but it appears to be 1000 to 10000 years (Schwarzschild and Härm 1970; Gingold 1974; Vassiliadis and Wood 1994); this evolution could result in a period *increase* or *decrease*. The time during which a star is in the instability strip during its transition from the AGB to the white dwarf stage is also a function of the properties of the star, but it appears to be 100 to 1000 years (Schönberner 1983; Percy *et al.* 1991); this evolution would result in a period *decrease*.

2. Period changes and evolution of RVT and SRd variables

Period changes in periodic variable stars can be studied using the (O–C) diagram, in which O represents the *observed* time of maximum or minimum brightness, and C represents the *calculated* (predicted) time, assuming the period to be constant. If the period is actually increasing or decreasing, rather than constant, then the (O–C) diagram will be a parabola, with positive or negative curvature (i.e., curving upward or curving downward), respectively. The characteristic rate of period change can be determined from the curvature of the parabola. The (O–C) in days is related to the time t in days by $(O-C) \sim \beta\, t^2/2P$ where β is the rate of period change in days/day, and P is the period (Percy *et al.* 1980; Willson 1986). The precision of β increases as the square of the length of the data set. We *arbitrarily* define the characteristic period-change time τ to be the time required for the period to change by *half* its own value, i.e., $P/2\beta$. So τ is 0.25 times the inverse of the coefficient of t^2.

This method can, in principle, be used to detect and measure the evolution of the star. If the radius R of the star is increasing, then the period will increase; if the radius is decreasing, then the period will decrease (the period is most strongly affected by the radius, and is approximately proportional to $R^{1.5}$). The quantity τ will then be a measure of the characteristic "evolution time" of the star.

There are two challenges to using this method: (i) The (O–C) diagram may be affected by random cycle-to-cycle period fluctuations, which cause it to deviate from a parabola. (ii) If there are gaps of many tens of cycles in the data set, the number of cycles in the gap may be uncertain, because of errors in measuring the times of maximum or minimum, and because of the random fluctuations, which produce wave-like excursions in the (O−C) diagram. As a result, there may be more than one possible interpretation of the (O−C) diagram, depending on how many cycles are assumed to fall in the gap.

Numerous studies of period changes in RV Tauri stars have been published, but Percy *et al.* (1997) showed that the (O−C) diagrams were dominated by random cycle-to-cycle period fluctuations of typically 0.005 to 0.02 of a period. These produce wave-like patterns in the (O−C) diagram which may masquerade as evolutionary period changes. As a result, the interpretation of the diagram may depend on the specific time interval involved. Percy *et al.* (1991) interpreted the (O−C) diagram of R Scuti in terms of an abrupt period change, whereas Matsuura *et al.* (2002), on the basis of 200 years of data, concluded that the period did not change—a conclusion that disagrees with post-AGB evolution, but not with blue-loop evolution.

3. Data

When we began this project in the autumn of 2003, AAVSO visual data were not yet all on-line, so we used the data from the Association Française des Observateurs d'Étoiles Variables (AFOEV 2003) and the Variable Star Observers League of Japan (VSOLJ 2003) which were available on-line. By late 2004, the AAVSO data were also on-line, so we have now used both data sets (AAVSO 2004) to determine the times of well-determined maxima and minima in the light curves using Hertzsprung's method. Since the minima tended to be better-defined than the maxima (a general property of RVT stars), we measured those preferentially, and used the times of maxima only when these were well-defined. Times of maximum were converted into equivalent times of minimum by adding a quantity which was determined for each star by averaging the interval between adjacent measured times of maximum and minimum. The interval (minimum–maximum) in days was 36 for AG Aur, 43 for AV Cyg, 66 for SX Her, 22 for UZ Oph, and 36 for TX Per.

We also measured light curves, or used already-determined times of maximum and minimum, taken from the astronomical literature (see list of references by Campbell, Gaposchkin, Gerasimovic, Huth, Jordan, Lacchini, Lause, Leiner, and Müller and Hartwig at the end of this paper). Although we have access to an excellent library, there were still observations and times of maximum and minimum in journals (mostly European) to which we did not have access, so this study is not the "last word" on the topic. Altogether, our data extended for most of a century for each star.

4. Results

We adopted periods and epochs (the time of the first observed maximum or minimum) for each of the stars (Table 1), and used these to calculate the values of (O–C) for each star. This required knowing the cycle number corresponding to each value. In most cases, the data were sufficiently continuous that we could determine the relative cycle numbers. In some cases, discussed below, the relative cycle numbers were ambiguous.

Table 1 contains five SRd/RVT variables, namely AG Aur, AV Cyg, SX Her, TX Per, and UZ Oph. These were investigated by Percy and Mohammed (2004) using self-correlation analysis, which determines the cycle-to-cycle behavior of the star, averaged over the data set. They found that AG Aur indeed showed some irregularity, AV Cyg and SX Per were quite regular despite their SRd classification, and UZ Oph and TX Per showed only "mild" RVT characteristics (see sections 4.1–4.5).

We adopted the periods given in the on-line version of the *General Catalogue of Variable Stars* (GCVS, Kholopov *et al.* 1985). For the SRd stars, these were simply the average intervals from adjacent maximum to maximum, or minimum to minimum. For the RVT variables, however, the periods are the intervals between adjacent *deep* minima.

Table 1. Characteristics of the five variables studied.

Name	*GCVS Type*	*P (days)*	*Length of Data Set (days)*	*tau (years)*
AG Aur	SRd	96.00	36601 (2415000–2451601)	+3980
AV Cyg	SRd	89.22	26189 (2426801–2452990)	−5900
SX Her	SRd	102.90	37112 (2415139–2452251)	+17500
UZ Oph	RVa (mild)	87.44	29899 (2422181–2452080)	−5560
TX Per	RVa (mild)	78.00	24907 (2427015–2451922)	−4100

4.1. AG Aur

The data are sparse between JD 2434093 and 2440196, so there are at least two possible interpretations of the (O–C) diagram. We have shown the two most likely ones in Figures 1 and 2; we consider Figure 1 to be the most likely.

4.2. AV Cyg

The data are sparse but reasonably well distributed, so we believe that Figure 3 is the correct interpretation. The scatter in Figure 3 may be due to the rather flat maxima and minima in this star.

4.3. SX Her

The (O–C) diagram is distinctly non-parabolic, presumably due to random cycle-to-cycle period fluctuations, so the best-fit parabola in Figure 4 may not represent the exact evolutionary trend.

4.4. UZ Oph

This star is a “mild” RVT variable in the sense that the secondary minima are shallow, and sometimes not discernible. There is some confusion about which are the deep minima, so we have used the half-period to construct the (O–C) diagram in Figure 5. The data are also sparse in places, so it is possible that there is a missing (or extra) cycle in one of the two larger gaps in Figure 5. In particular: readers may note that the later points in Figure 5 could be raised by half a cycle to produce a flatter (O–C) diagram, in which case the characteristic time τ would be −22900 instead of −5560 years. We chose the interpretation in Figure 5 because of the trends of the points just before and after the gap.

4.5. TX Per

This star is a “mild” RVT variable in the sense that the minima, which are 78 days apart, are very similar in depth. The data are sparse in the middle of Figure 6, so it is possible that there is a missing (or extra) cycle, though the beginning and end of the gap connect well. There may be a miscounted cycle between the first and the second point, but this will have a negligible effect on the result.

5. Random cycle-to-cycle period fluctuations in SRd variables?

Eddington and Plakidis (1929) showed that some Mira variables show random cycle-to-cycle period fluctuations, which dominate the (O–C) diagram in most of these stars. Percy and Colivas (1999) found such fluctuations in most of a sample of almost 400 bright Miras in the AAVSO visual observing program; the typical fluctuation was about 0.01 to 0.05 of the period. Percy *et al.* (1997) also found such fluctuations in 15 RV Tauri stars; the typical fluctuation was about 0.005 to 0.02 of the period.

The wave-like patterns in the (O–C) diagrams of the 5 RVT/SRd variables (Figures 1–6) are suggestive of the effect of random period fluctuations. However, our preliminary analysis suggests that, although the cycle-to-cycle fluctuations are probably present, they are masked by unusually large errors in measuring the times of maximum and minimum. This is likely because of the very heterogeneous nature of the measured times which we have used.

6. Discussion and conclusions

The five stars studied in this paper show (O–C) diagrams which can be interpreted in terms of period changes which are not inconsistent with models of stellar evolution. We cannot make a stronger statement because the (O–C) diagrams appear to be dominated by random cycle-to-cycle period fluctuations; some or all of the curvature in the (O–C) diagrams may be due to the wave-like patterns caused by these fluctuations. The measured curvatures do, however, provide reasonable *upper limits* to the rate of period change, or lower limits to the evolutionary time scales.

Despite the efforts of visual observers over the past century, there are still some gaps in the (O–C) diagrams, though these might possibly be filled in by data which are presently not available to us. We encourage visual observers to continue to monitor these stars, since our understanding of their period changes increases as the *square* of the length of the data set.

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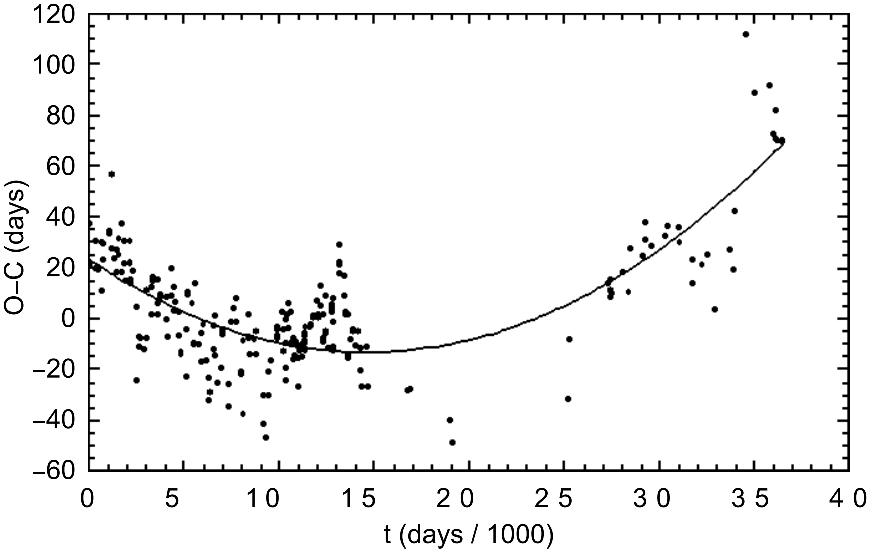

Figure 1. The most likely (O–C) diagram for AG Aur, based on a century of visual observations. The line is the best-fitting parabola. Compare with Figure 2.

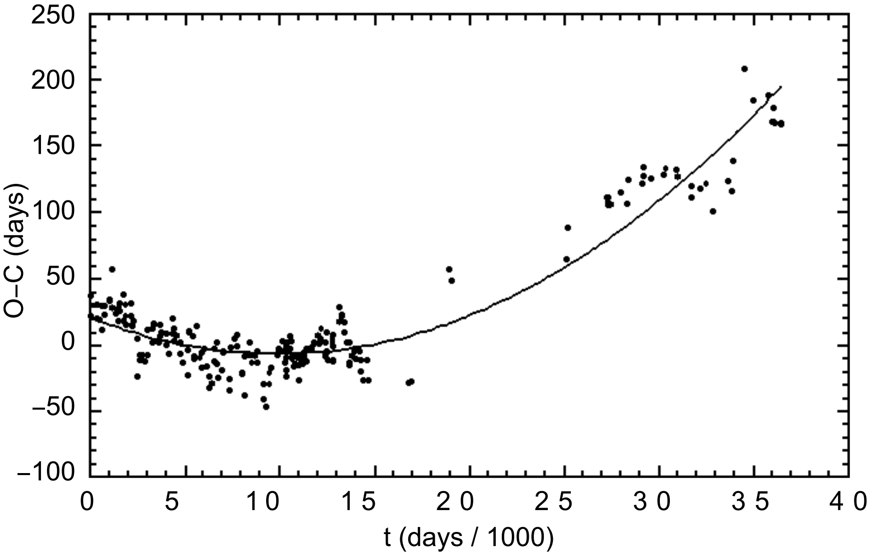

Figure 2. A less-likely (O–C) diagram for AG Aur; an extra cycle has been included at n/1000 = 18. The line is the best-fitting parabola.

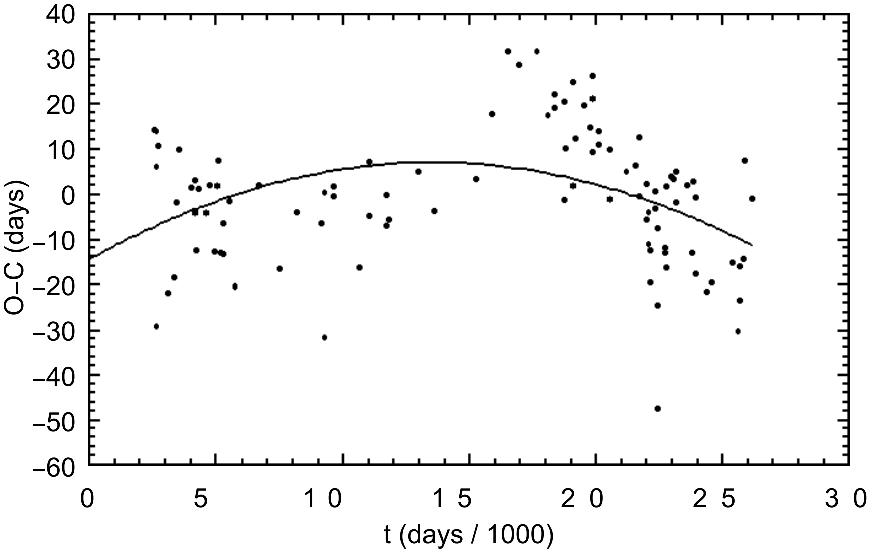

Figure 3. The (O–C) diagram for AV Cyg, based on 72 years of visual observations. The line is the best-fitting parabola, but the diagram is dominated by the wave-like pattern caused by random cycle-to-cycle period fluctuations.

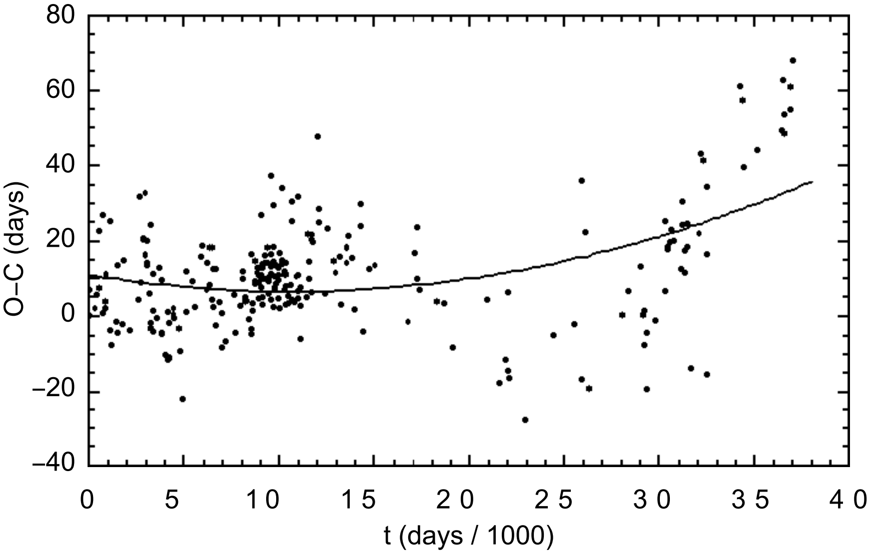

Figure 4. The (O–C) diagram for SX Her, based on over a century of visual observations. The line is the best-fitting parabola, but the diagram is dominated by the wave-like pattern caused by random cycle-to-cycle period fluctuations.

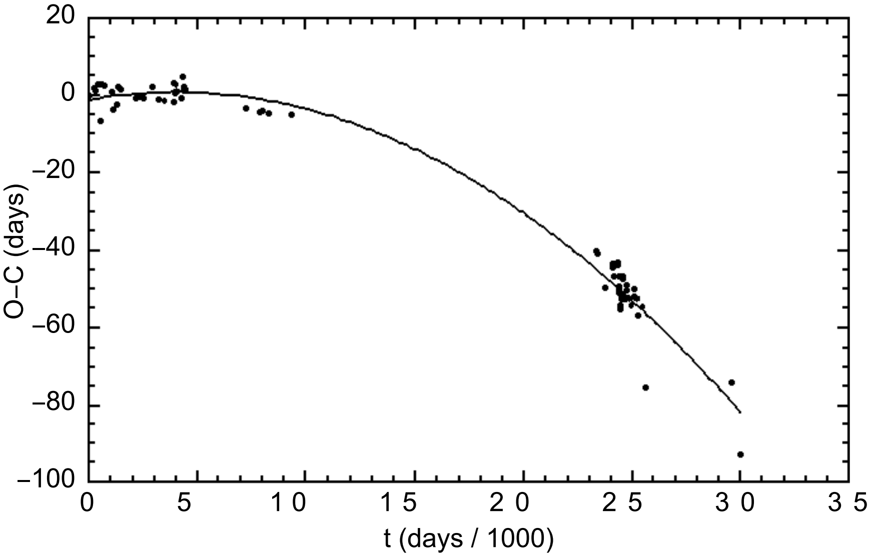

Figure 5. The (O–C) diagram for UZ Oph, based on 82 years of visual observations. The line is the best-fitting parabola. Because of the large gap in the data, it is possible that there is an error in the number of cycles in the gap; see comment in the text.

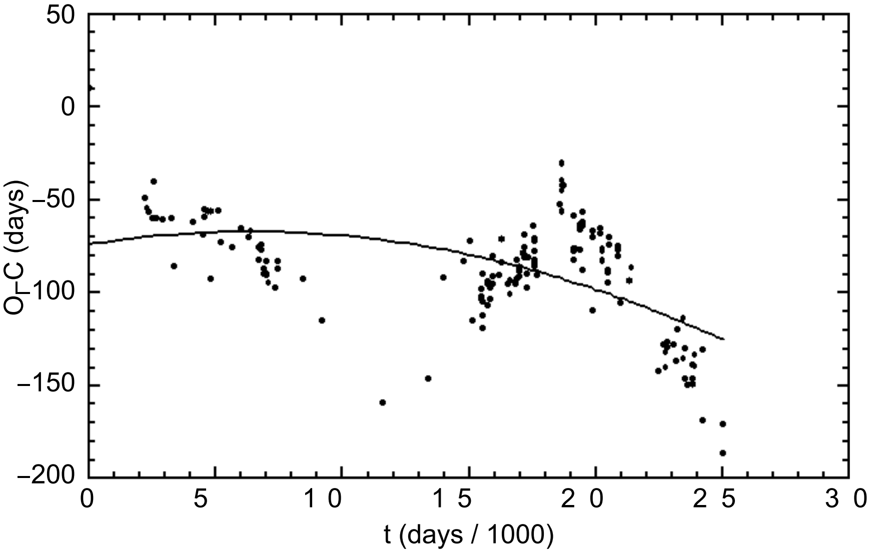

Figure 6. The (O–C) diagram for TX Per, based on 68 years of visual observations. The line is the best-fitting parabola, but the diagram is dominated by the wave-like pattern caused by random cycle-to-cycle period fluctuations.