Periodic phenomena
&
O-C diagrams

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Really boiled down version:

- We'll assume our variable star changes brightness with a pattern that repeats at regular intervals.
- Does it really?

Overview

- In nature, many phenomena repeat at regular intervals – they are *periodic*
- It is natural to try to learn the period of repetition, and whether it changes
- So too, for variable stars: many types are periodic, due to wildly different physical phenomena. Changes in the periods further reveal processes in those star systems, and the interiors of those stars.
- AAVSO observers' data can contribute to those studies
- We will discuss aspects of this, using eclipsing binary stars as examples

Outline:

Cartoon history of studying the period of a variable star:

- Notice different brightness on different dates
- Start recording brightnesses
- Get a sense of min and max possible period
- Make a period estimate, and choose a reference time
- Start predicting times of min or max brightness
- Observe actual times of min or max brightness *
- Refine period estimate and thence predictions
- Continue observing
- Monitor to see if period is constant, or changing (O-C)
- Predict future period changes…
- Analyze the physical causes of the period changes…

Definitions:

- Light curve (LC): a graph showing the brightness of a star versus time.
- Phased light curve: a LC over just one period (sometimes 2), showing the representative behaviour of the star over one cycle.
- Period: the length time after which the same phase occurs again. E.g., Time between two successive LC maxima, or the time interval between two successive primary eclipses.
- ToM, time of minimum (or maximum): the centre point of a min or max on the lightcurve. For eclipsing binaries, the ToM is taken to be the time of the eclipse.

Use Julian dates & heliocentric correction:

- What is the time interval between:

  August 24th, 2021, 4:46 pm MDT, and
  April 14th, 1973, 2:12 pm EST

- Vastly easier to use Julian dates:

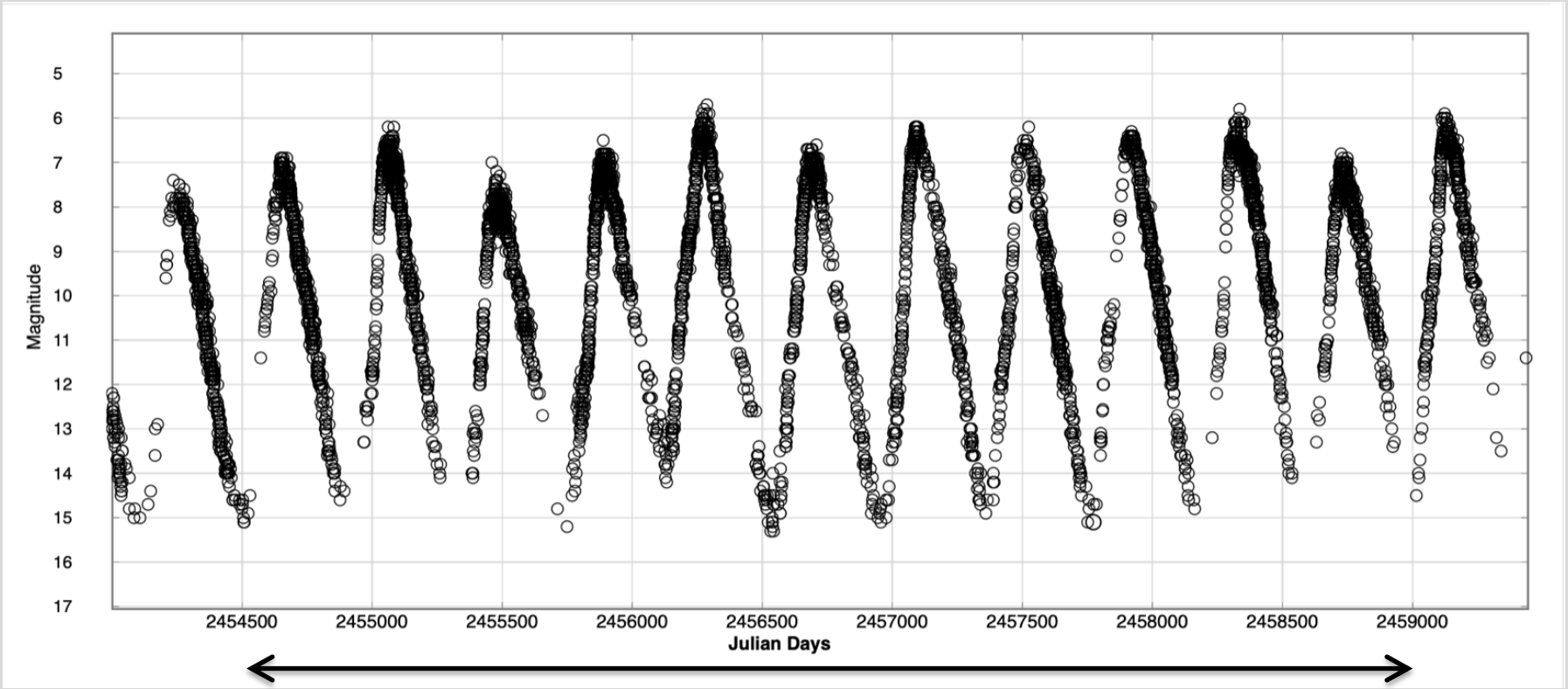
```
   245 9451.4486
 - 245 1787.2583
   -------------
 =      7664.1903 days
```

- As earth orbits the sun, it alternately becomes up to 500 light-seconds nearer and further from distant objects. This is much greater than the precision of our measurements, and of phenomena we want to study – must correct for it giving “HJD”.

Example - a long period variable star:

Observation interval much less than the period.

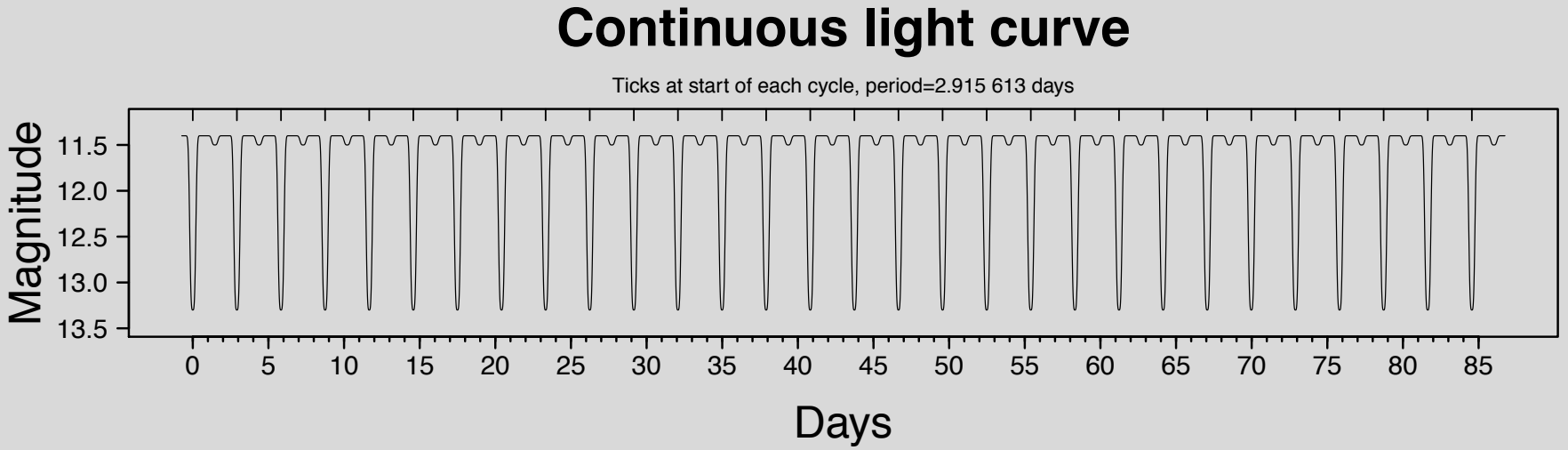
R Andromedae, 15 years of AAVSO data

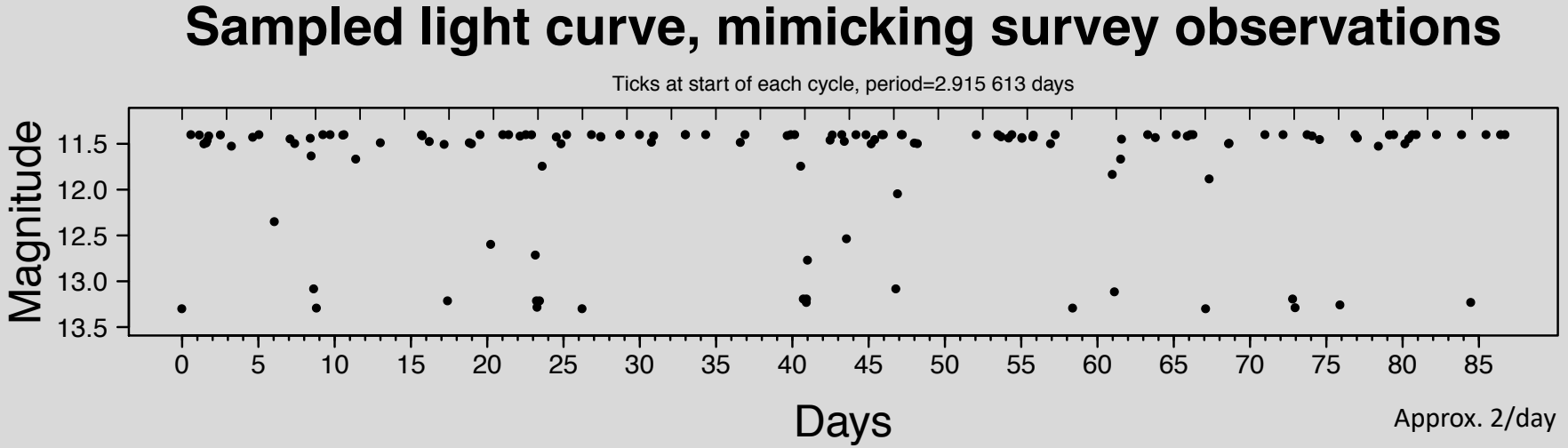


4500 days, 11 cycles: period ~409.1 days/cycle

Example - a short period variable star:

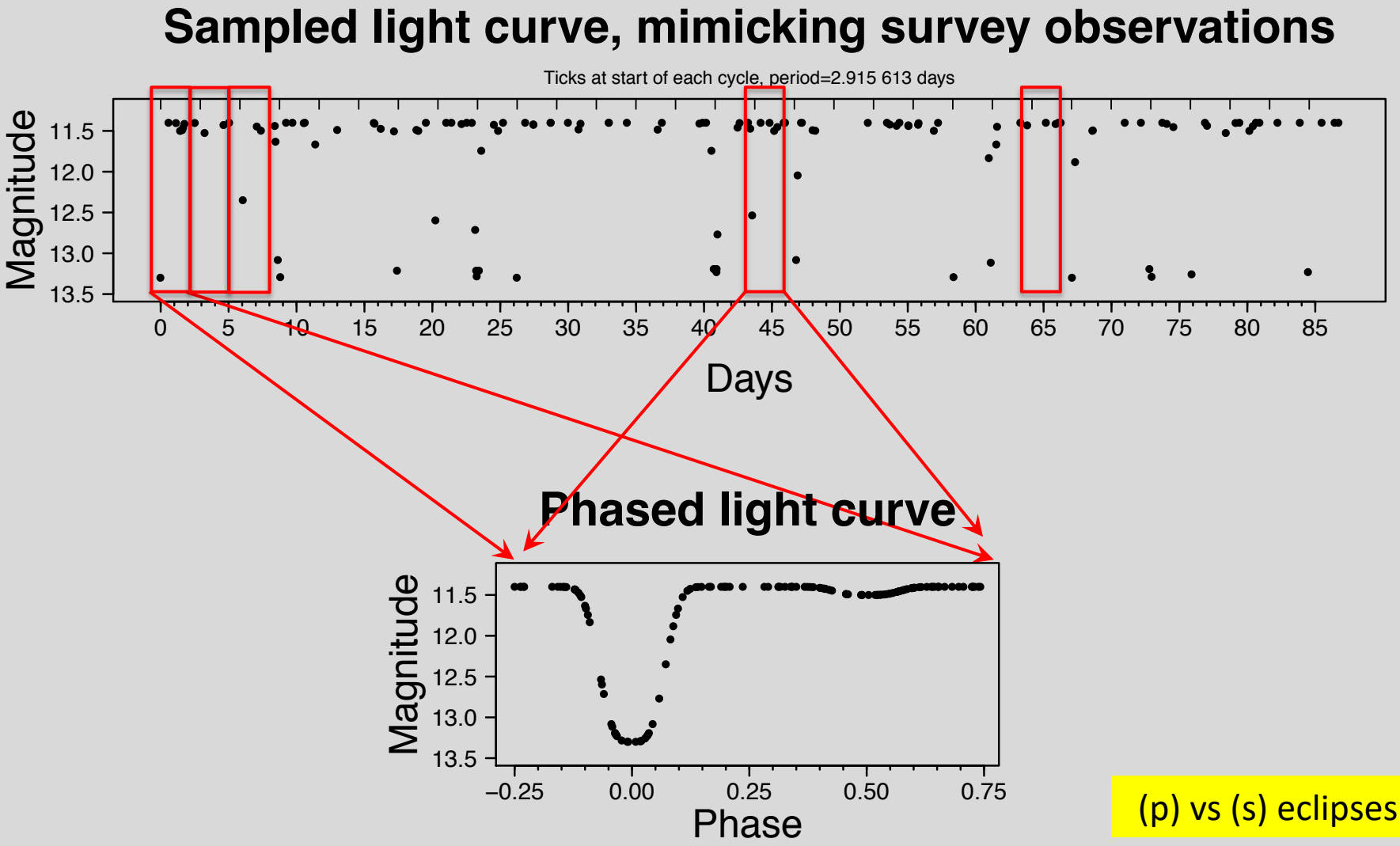
Synthetic EB light curve, simulating LS Persei





Approx. 2/day

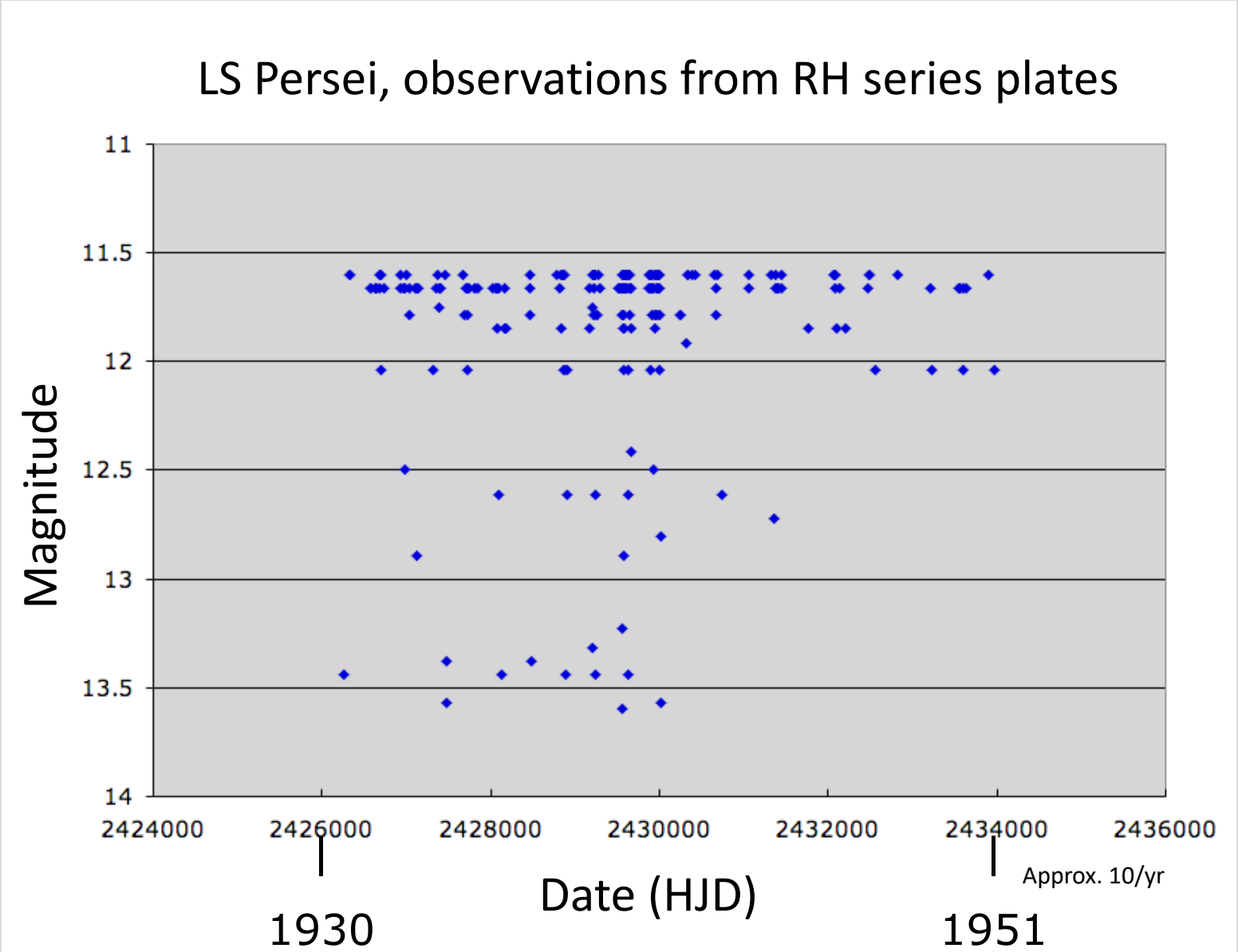
Synthetic light curve, closely matching LS Per:



(p) vs (s) eclipses

Real survey observations of LS Per:

Randomly timed observations over two decades.



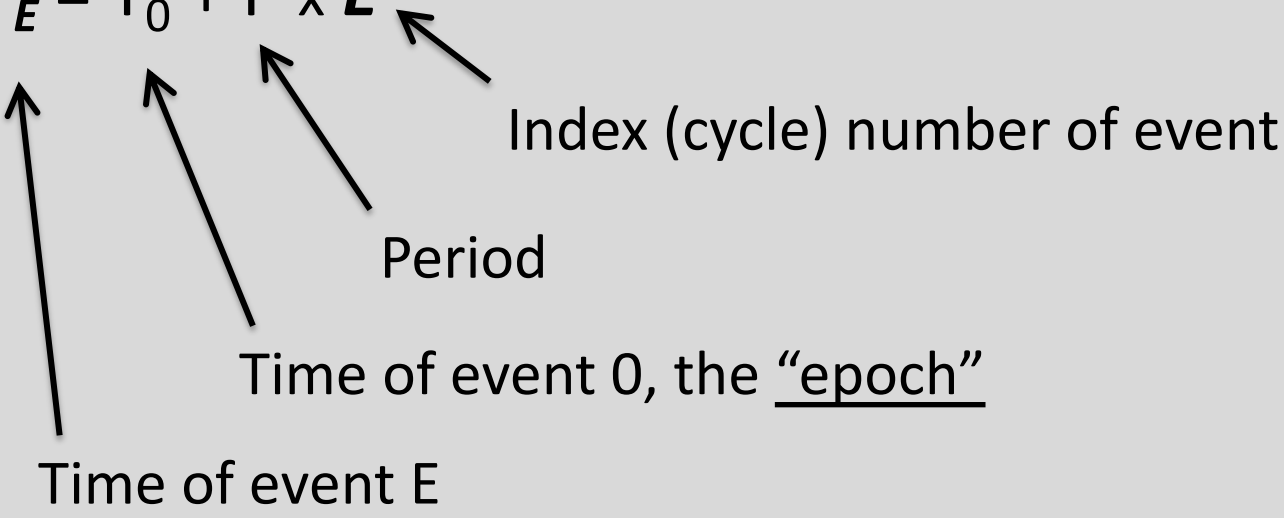
Describing periodic phenomena:

Based on the equation of a straight line:

$$y = mx + b$$

$$T_E = PE + T_0$$

$$T_E = T_0 + P \times E$$



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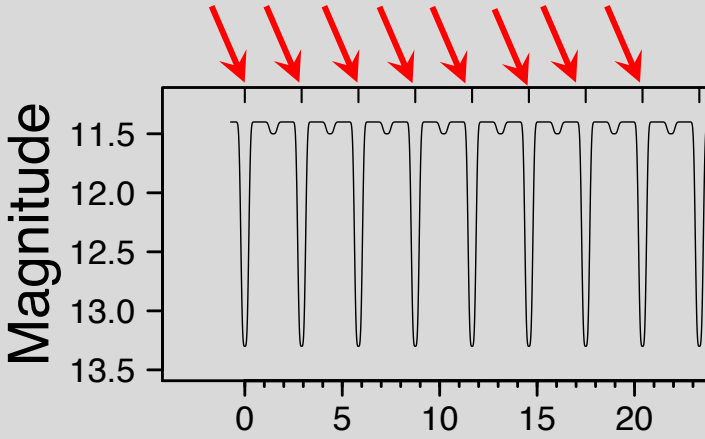


$$T_E = PE + T_0$$

$$T_E = T_0 + P \times E$$

Index (cycle) number of event

Period

Time of event 0, the "epoch"

Time of event E

Basics of periodic phenomena:

- A real example (LS Persei):

$$T_E = T_0 + P \times E$$

(Heliocentric) Julian date of the eclipse that started cycle 0

Length of each cycle – the "period"

- Filling in the numbers, LS Per's eclipses are predicted by the equation:

$$T_E = 243\,9395.52 + 2.915\,693 \times E$$

Cycle number

The "elements"

- This equation predicts the start time of each cycle (for EBs, the middle of the deepest eclipse)

- To make a phased light curve so you can readily compute where, within a cycle, an observation occurred... then plot observations from many cycles as if they were all observed within one cycle.

Basics of periodic phenomena:

- O-C is done after a (pretty) good period estimate has been made.
- Must determine the cycle number (E):

  $\mathbf{T_E} = T_0 + P \times \mathbf{E}$

  $\mathbf{E} = (\mathbf{T_E} - T_0) / P$
- E is usually used as a whole number, so must be rounded (usually down)
  - Can be a bit tricky if T_E-T_0 is large (say, > P/2)
  - Well suited to a simple spreadsheet

- The whole and fractional cycles:

  $\mathbf{E} = round[\ (\mathbf{T_E} - T_0)/P\ ]$

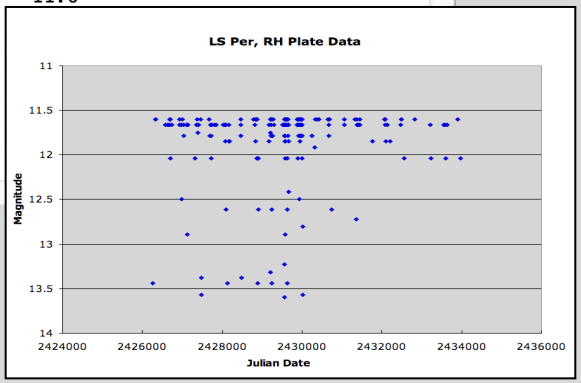
  $\phi = fractional\_part[\ (\mathbf{T_E} - T_0)/P\ ]$ a.k.a “phase”

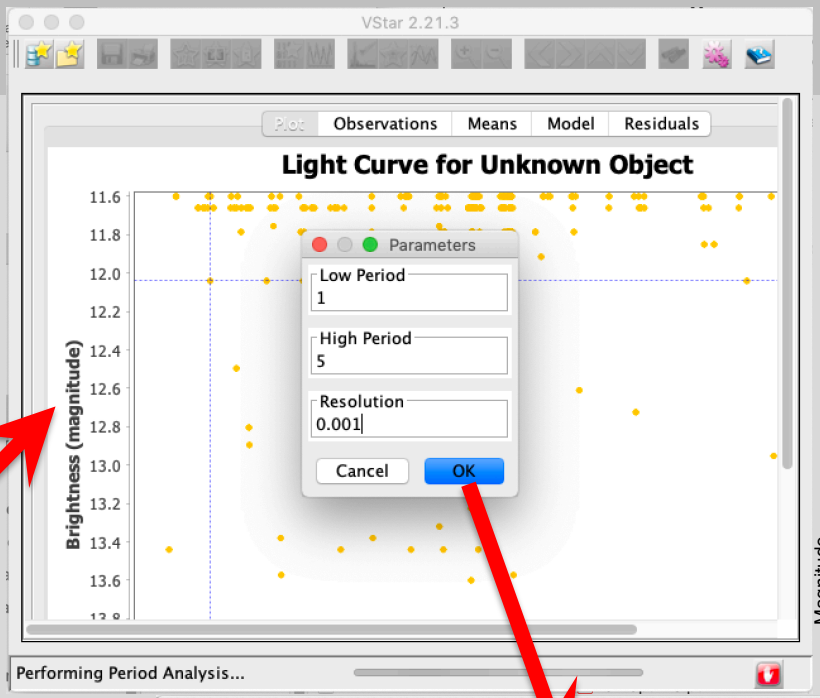
Period estimation:

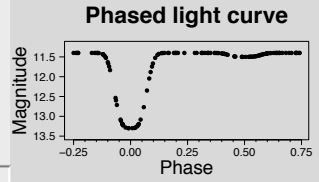
Using “VStar”:

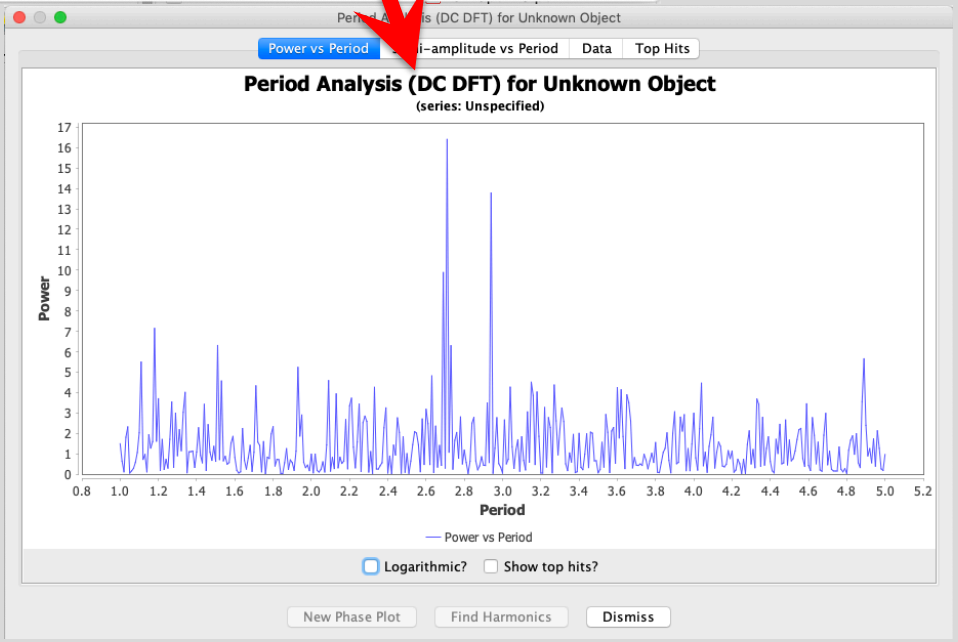
HCOmagsRH.txt

2427671.838	11.6
2428832.714	11.6
2429555.687	11.6
2429613.611	11.6
2429626.539	11.6
2429626.579	11.6
2429940.68	11.6
2429983.561	11.6
2429988.59	11.6
2430389.589	11.6
2433899.805	11.6
2426328.519	11.6
2426331.608	11.6
2426680.841	11.6
2426706.504	11.6
2426932.846	11.6
2427002.645	11.6
2427372.682	11.6
2427464.521	11.6
2428466.753	11.6
2428782.844	11.6
2428832.753	
2428847.714	
2428868.619	
2428873.621	
2429197.711	
2429217.585	
2429224.652	
2429234.571	

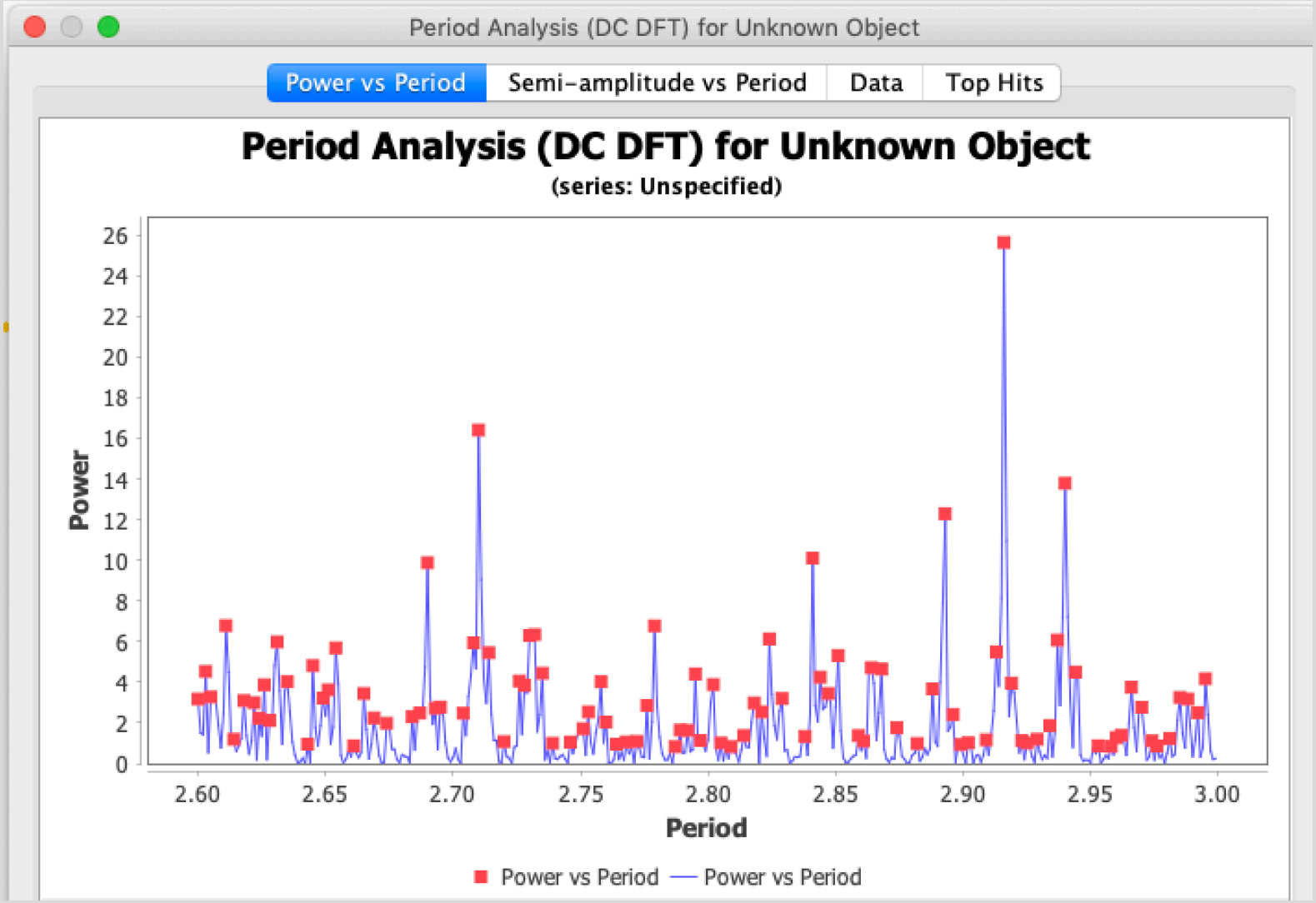








Basics of periodic phenomena:



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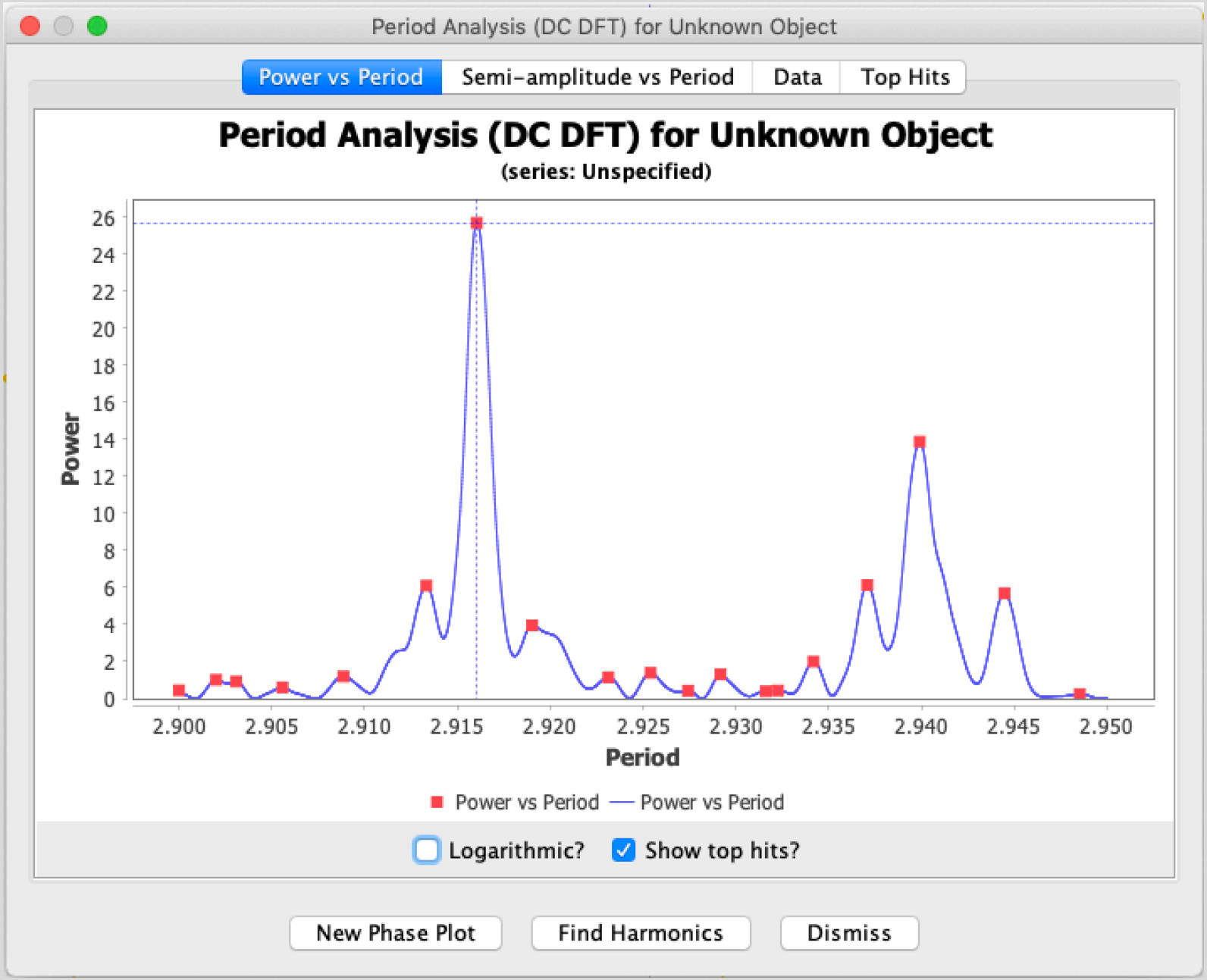
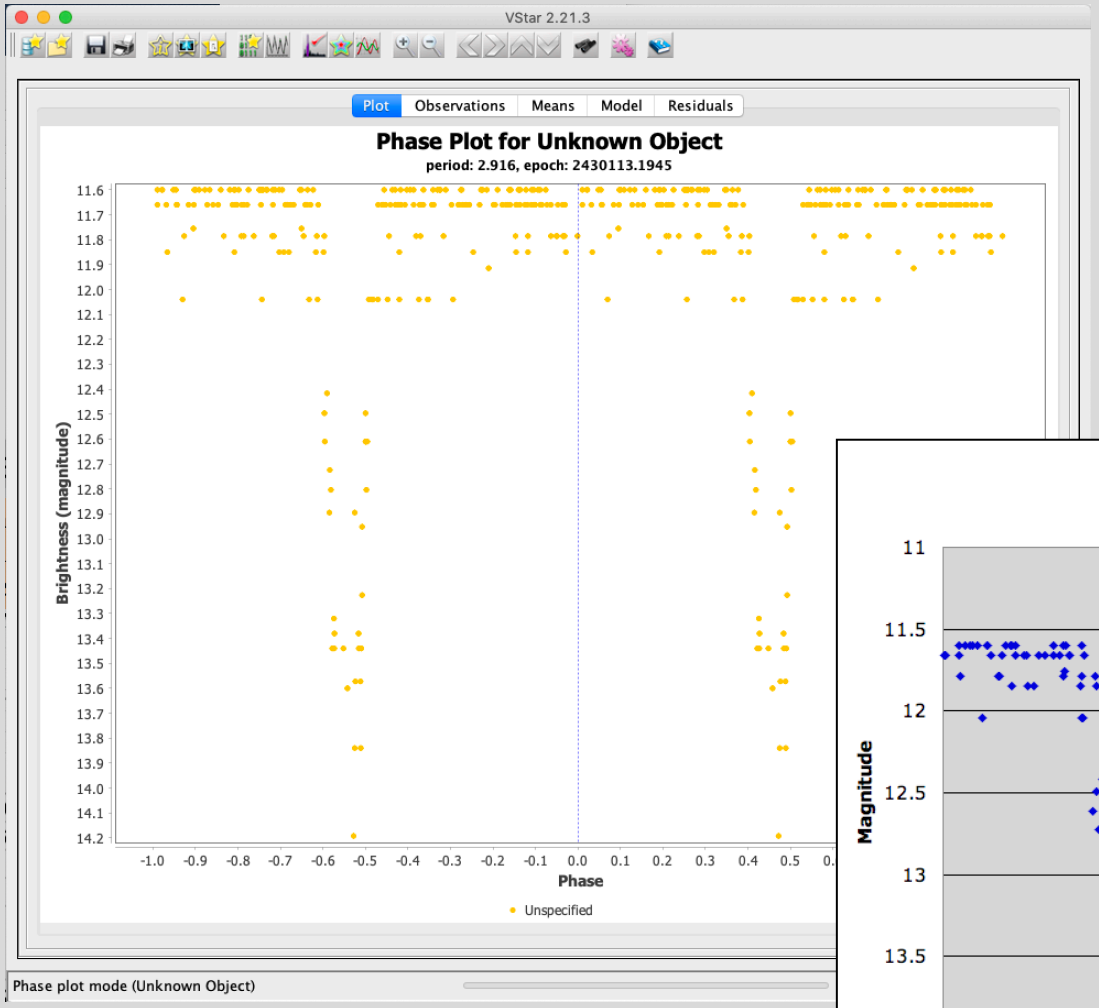
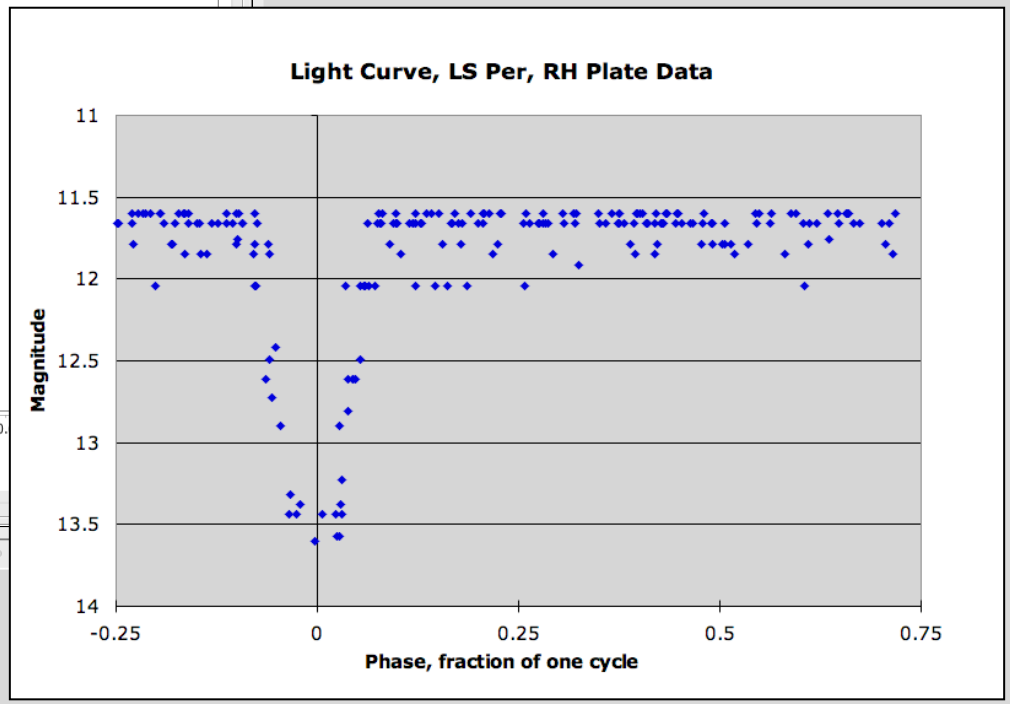
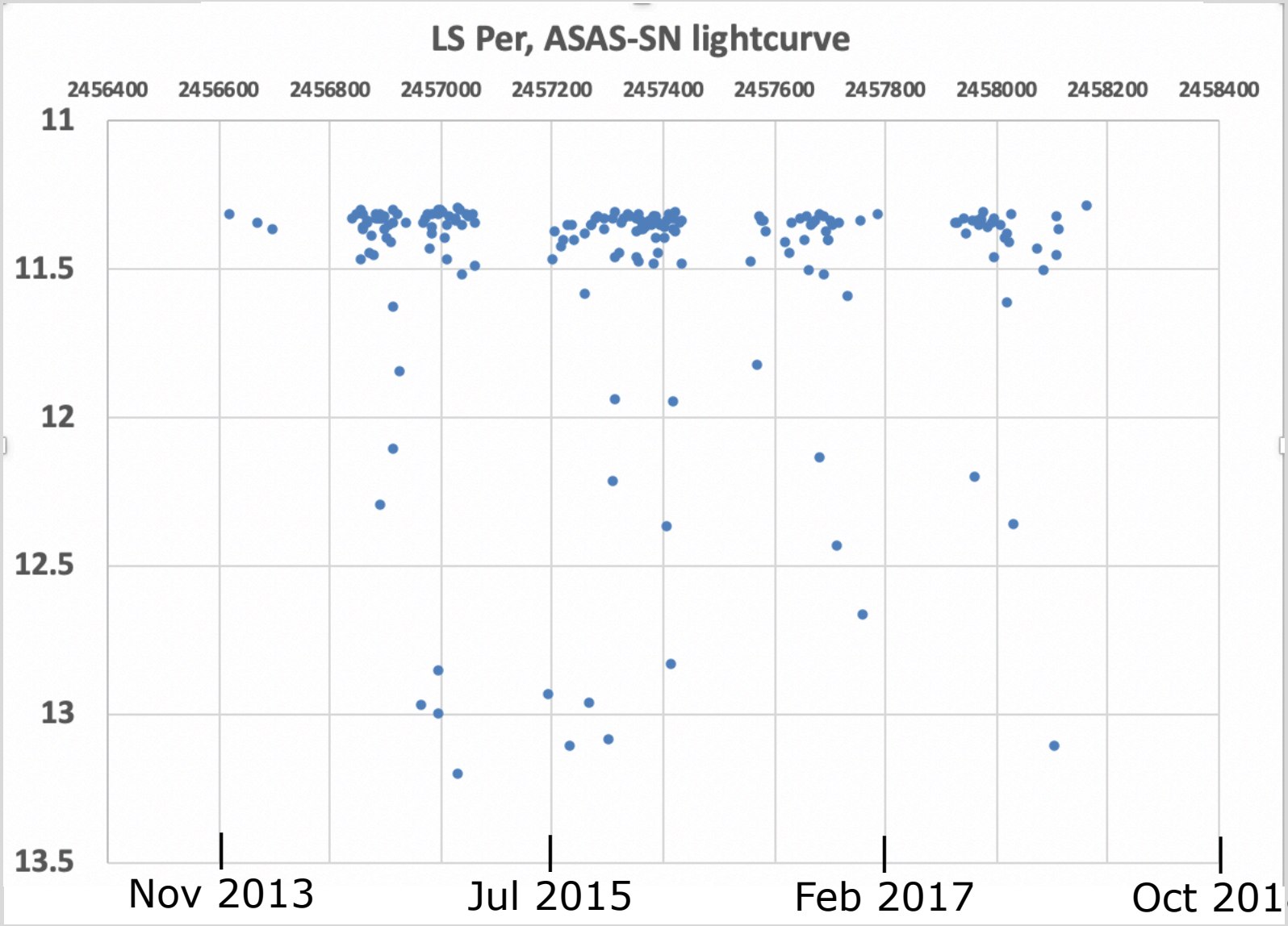


Period estimation:





Recent survey data for LS Per:



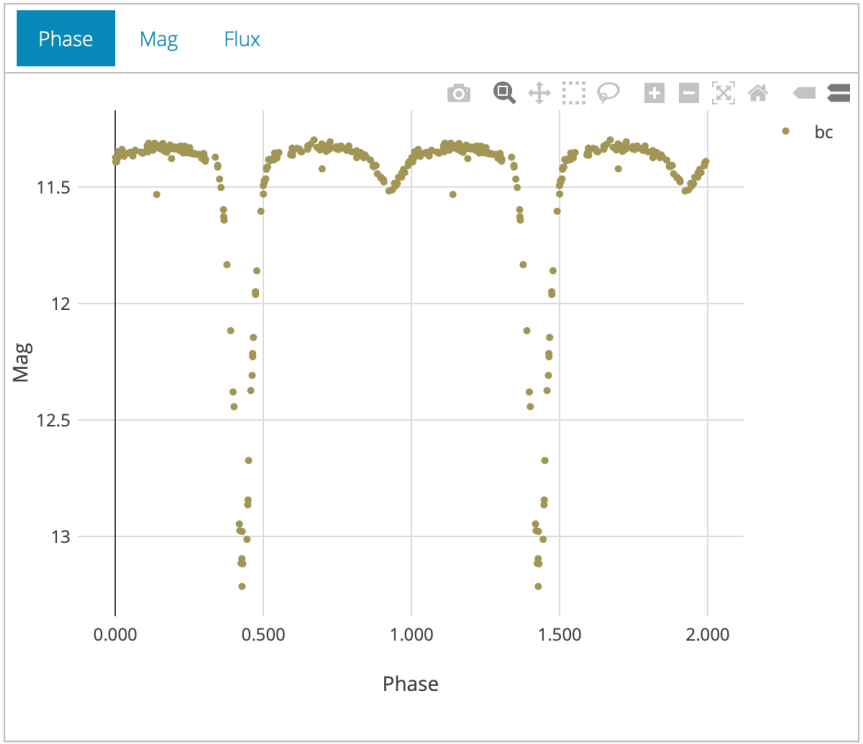
Recent survey data for LS Per:

ASAS-SN Sky Patrol | Variable Stars Database | Variable Stars Atlas | Photometry Database

ASASSN-V J025708.79+375340.8 / LS Per (44.28664, 37.89467)

Mean VMag	Amplitude	Period	Type
11.51	1.69	2.9155441	EA

Classification Probability	LKSL Statistic	RFR Score	Epoch (HJD)
0.998	0.02	0.99	2458110.69759



Adjust Period

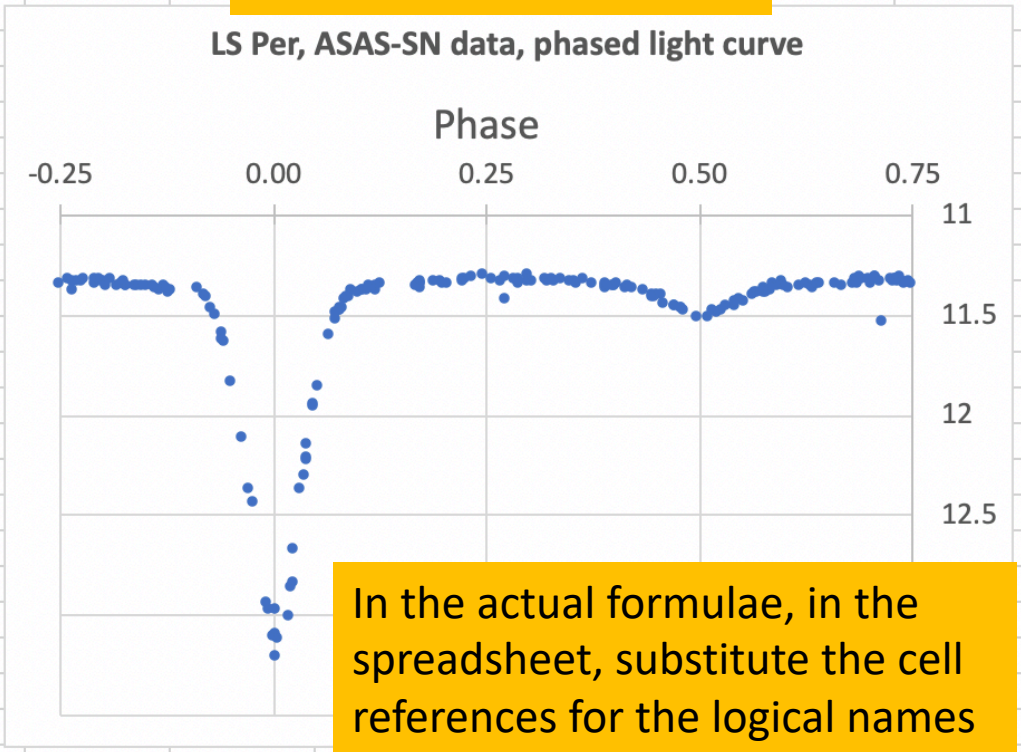
Submit | 1/2 Period | 2x Period | Reset

Making a phased LC in a spreadsheet:

The input data: HJD & mag

	A	B	C	D
1	LS Per, ASAS-SN photometry			
2				
3		epoch	2458110.698	
4		period	2.9155441	
5				
6	HJD	phs	wrapped	mag
7	2457977.086	0.172739147	0.1727	11.351
8	2457970.021	0.749452186	0.7495	11.351
9	2457289.067	0.189513923	0.1895	11.334
10	2457428.751	0.099675872	0.0997	11.387
11	2457721.816	0.61766495	0.6177	11.355
12	2456896.053	0.39009861	0.3901	11.344
13	2458029.86	0.273634962	0.2736	11.422
14	2457417.759	0.329679733	0.3297	11.339
15	2457001.942	0.708726066	0.7087	11.315
16	2457265.017	0.940569927	-0.0594	11.597
17	2457364.905	0.20118701	0.2012	11.348
18	2456890.022	0.321410985	0.3214	11.327
19	2457422.772	0.049043196	0.0490	11.961
20	2457285.021	0.801975316	-0.1980	11.341
21	2458014.074	0.859167042	-0.140	11.36
22	2457440.719	0.204638647	0.2046	11.349
23	2456888.044	0.642933852	0.6429	11.342
24	2457332.903	0.224752114	0.2248	11.338
25	2456904.016	0.121238914	0.1212	11.375
26	2457961.089	0.685783899	0.6858	11.349

Plot columns “mag” vs “wrapped”.



In the actual formulae, in the spreadsheet, substitute the cell references for the logical names I have used below.

Compute phase, using Excel formula:
=MOD(HJD-epoch,period)/period

Make phase go from -0.25 to 0.75, instead of 0 to 1, using Excel formula:
=if(phs>0.75,phs-1,phs)

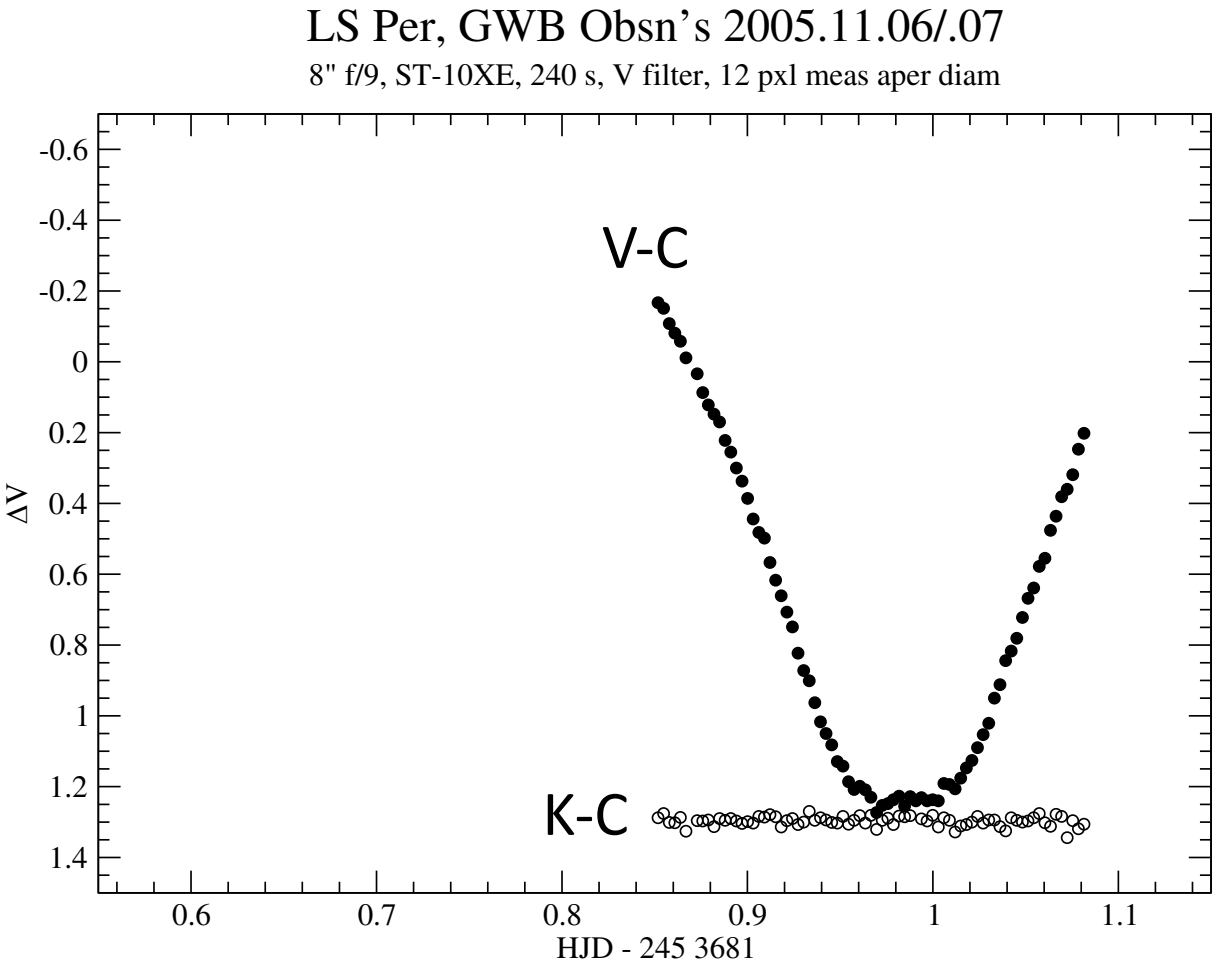
Where are we in the story?

- We are up to where most EB observers start: we have a period estimate, and a reference ToM (T_0) – the "light elements".

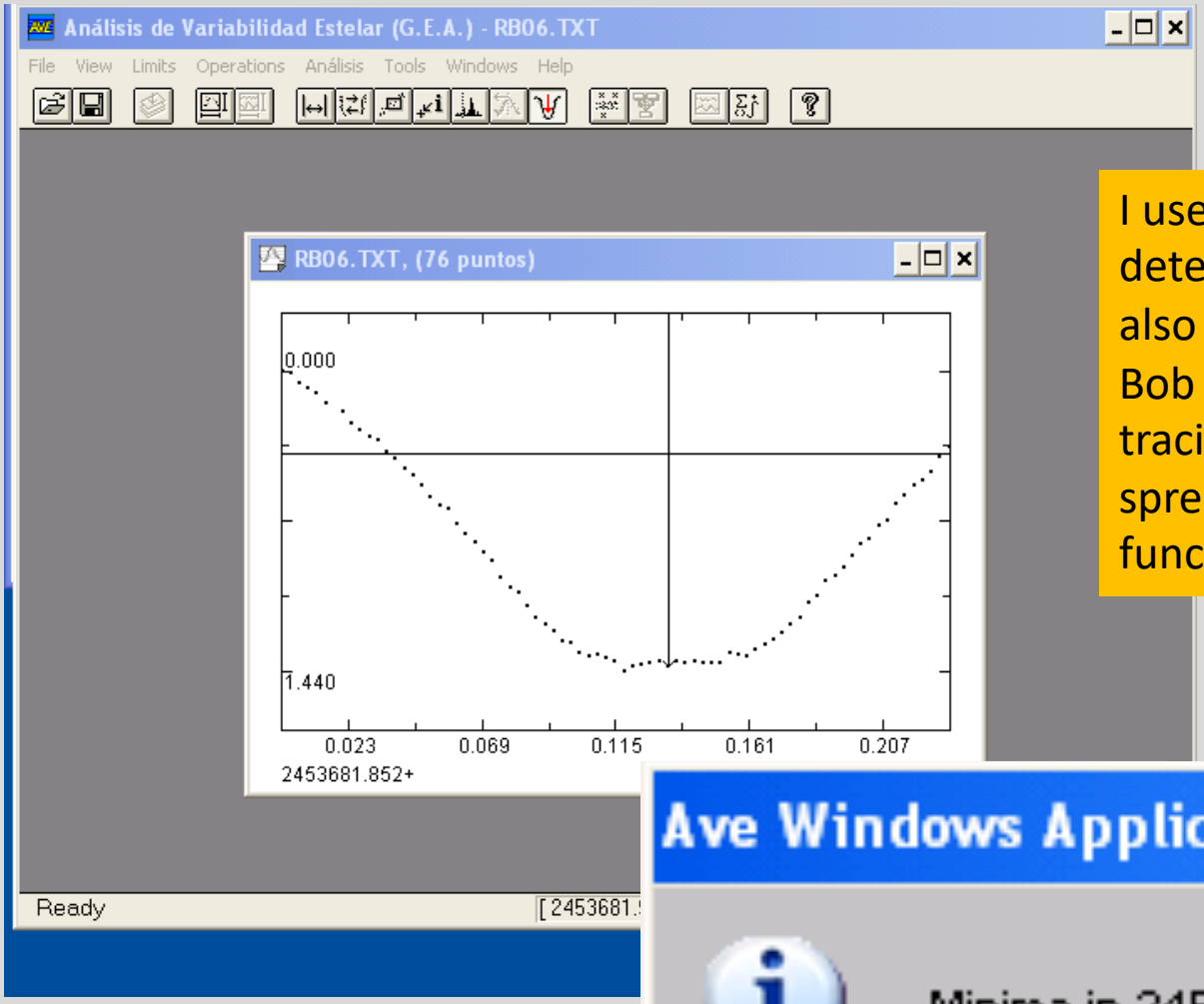
- Use the ephem eq'n to ***calculate*** or predict ToMs, then ***observe*** actual ToMs with time-series observations through individual eclipses

CCD observations through an eclipse:

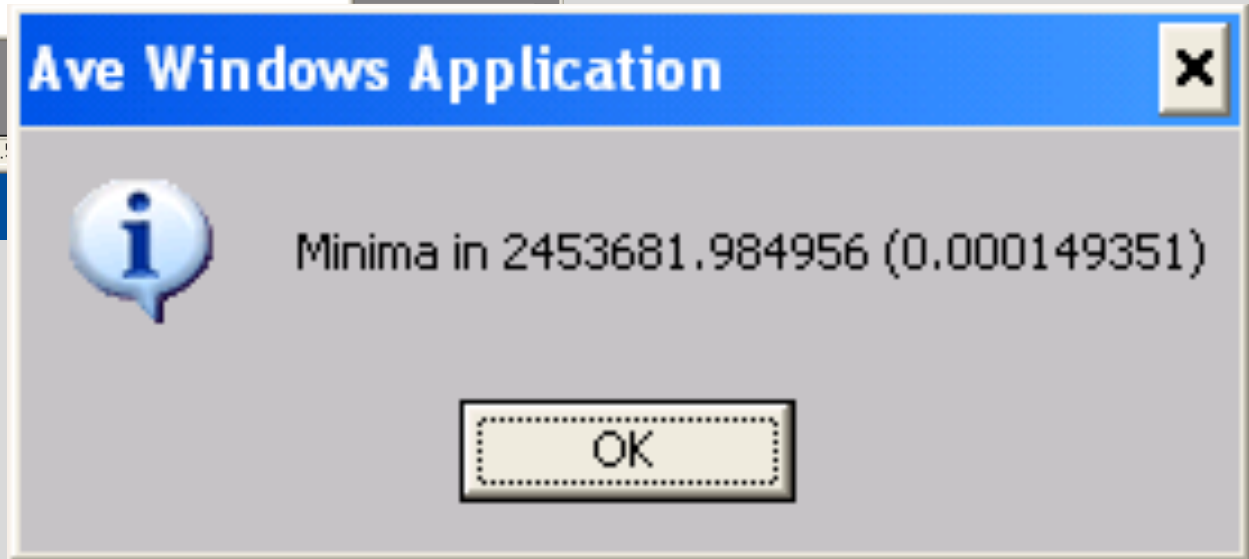
Targeted, intense, observing run gives high precision ToM:



Determine a ToM from an intense time series:



I used a program "AVE" to determine the ToM. Peranso can also do this, as can a program by Bob Nelson. Can also do "digital tracing paper method" in a spreadsheet, fit a symmetric function in R, etc.



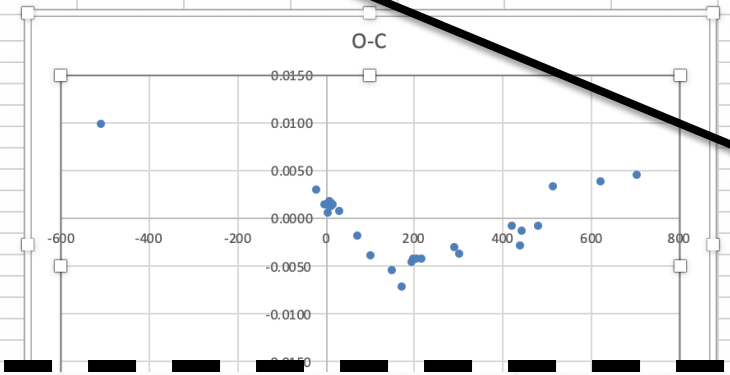
Calculate period from ToMs:

```
  245 9275.9754   Cycle  5859.0  (March 2021)
- 245 1978.9716   Cycle  - 33.5  (March 2001)

    7297.0038  days  / 5825.5 cycles

               = 1.252597 days/cycle
```

For more than 2 ToMs, use, e.g., Excel's LINEST function to fit a straight line to your ToMs. The fitted slope is P, the intercept is T_0.

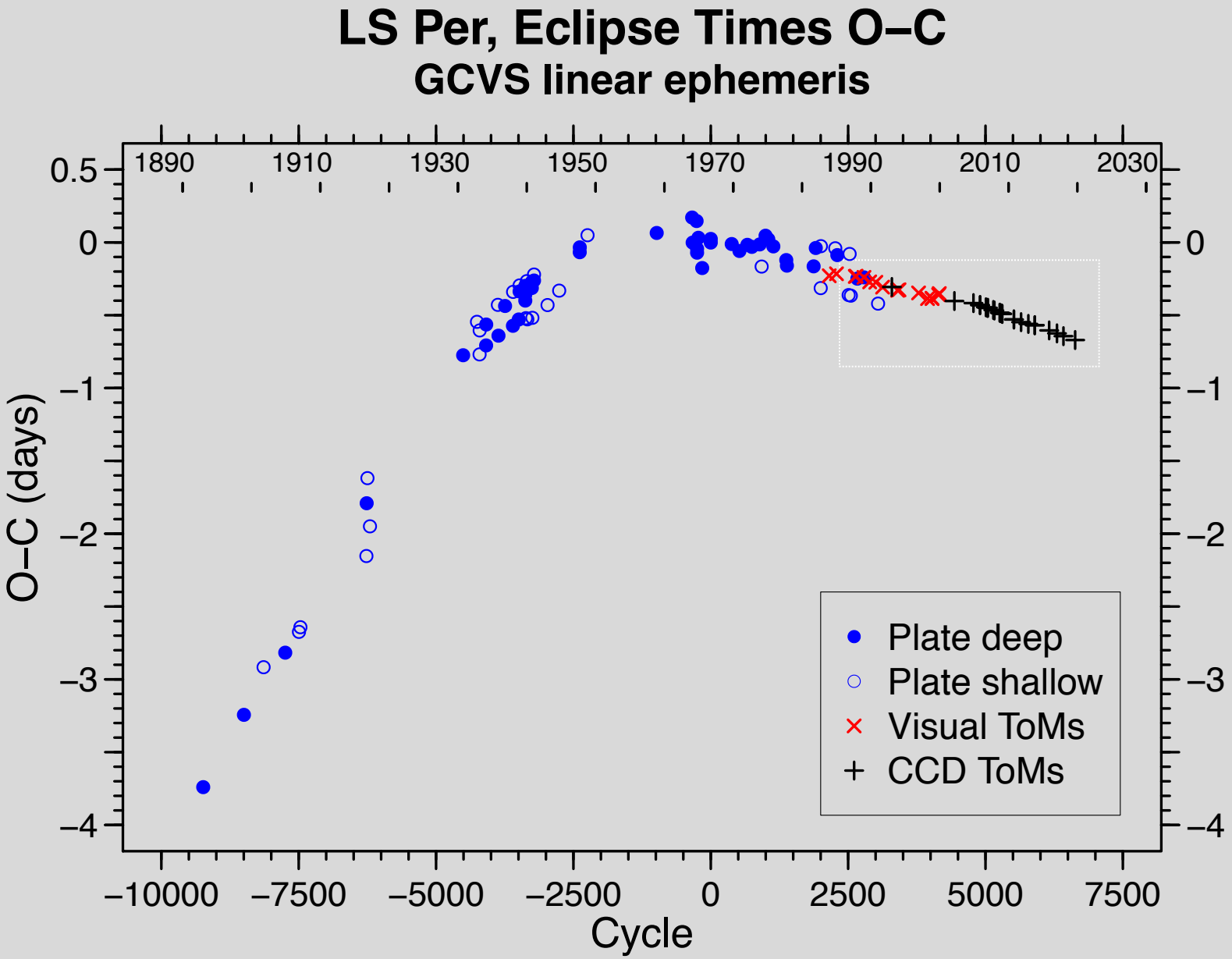
Calculate O-C in a spreadsheet:

	A	B	C	D
1	epoch	2451937.009		
2	period	1.2525966		
3				
4				
5	cycle	Observed ToM	Predicted ToM	O-C
6	-15.5	2451917.5965	2451917.594	0.0027
7	-504	2451305.7100	2451305.7	0.0097
8	-489.5	2451323.8400	2451323.863	-0.0230
9	0	2451937.0103	2451937.009	0.0013
10	4	2451942.0206	2451942.019	0.0012
11	8	2451947.0301	2451947.03	0.0003
12	13.5	2451953.9207	2451953.919	0.0016
13	16	2451957.0516	2451957.051	0.0011
14	17.5	2451958.9309	2451958.929	0.0015
15	21.5	2451963.9411	2451963.94	0.0013
16	33.5	2451978.9716	2451978.971	0.0006
17	76.5	2452032.8306	2452032.833	-0.0020
18	106	2452069.78011	2452069.784	-0.0041
19	155.5	2452131.7821	2452131.788	-0.0057
20	177	2452158.7113	2452158.719	-0.0073
21	197	2452183.7657	2452183.771	-0.0048
22	201	2452188.7764	2452188.781	-0.0045
23	210.5	245220		
24	222.5	2452215		
25	297	245230		
26	306.5	245232		
27	427	245247		
28	444.5	2452493		
29	448.5	2452498		
30	486	245254		
31	517	245258		
32	625	2452719		
33	708	2452823		
34	720	245283		
35	729.5	245285		
36	745.5	2452870		
37	749.5	2452875		
38	755	245288		
39	896.5	245305		
40	922	2453091		



Making an O-C diagram in a spreadsheet

O-C, with new data:



End of Part 1:

In the next talk, I will carry on from this point, and:

- Examine whether our light elements accurately fit our ToMs, and
- test whether the “equation of a straight line” formula is sufficient to fit our data.

Resources re O-C:

General information:

- Sterken, C., (editor) (2005). "The light-time effect in astrophysics. Causes and cures of the O-C diagram". ASP Conference Proceedings Volume 335. The first chapter (by Sterken) is a good overview, and is available via ADS as 2005ASPC..335….3S There are many other useful papers in this volume.
- Budding, Edwin, and Osman Demircan (2007). "Introduction to astronomical photometry". Cambridge Univ. Press. This book is much broader than its title suggests. Chapter 8 (28 pages) is titled "Period changes in variable stars" and discusses the phenomena described in this presentation, in more depth. It also includes lists of stars that exhibit these phenomena, and a useful annotated bibliography.
- BRNO O-C Gateway: http://var2.astro.cz/ocgate/?lang=en. Access to an online database of EB ToMs, and plotting an O-C with parameters the user can control. You can also add your own datapoints to the plot, or download their data for plotting yourself. Includes more stars than the Lichtenknecker Database.
- Other online O-C resources:
  - Bob Nelson has compiled ToM data for thousands of EBs and makes it available as spreadsheets for each star, along with plotting and observation planning capability: https://www.aavso.org/bob-nelsons-o-c-files
  - "The Cracow Database" http://www.as.up.krakow.pl/ephem/ Predictions of upcoming eclipses (when the orbits are well enough known; for planning observations), and O-C diagrams for thousands of EBs.

Technical material for modelling complex cases:

- Lacy, C.H.S. (1992), AJ 104(6):221. For modelling systems with apsidal motion.
- Irwin, J.B. (1952). ApJ 116:211-217. For computing 3rd body orbits.
- Mayer, P. (1990). Bull. Astron. Inss. Czechosl. 41:231-236. Re computing the mass function using O-C quantities (versus radial velocity data).
- Zavala, R.T. et al. (2002). AJ 123(Jan 2002): 450-457. Regarding multiple effects in the O-C diagram for WW Cyg. A gateway paper to considering the Applegate mechanism of cyclic variations.
- Frieboes-Conde, H., and T. Herczeg (1973). Astron. Astrophys. Suppl. 12:1-78. Re screening criteria for detecting LTE systems.

A few stars with interesting O-C diagrams:

- Algol, beta Per – for its historical interest. Multiple effects.
- CL Aur: 3rd body LTE and long term period increase due to mass transfer.
- SZ Cam: 2.7 d EB, with 51 yr 3rd body LTE (eccentric orbit)
- DR Vul: apsidal motion.

One source for recent photometric survey data:

- ASAS-SN data, by variable star name: https://asas-sn.osu.edu/variables/lookup?utf8=✓